Physics 101H

General Physics 1 – Honors

Lecture 31 – 10/31/22

Newton's law of gravitation

Summary

Topics

Friday: Deformable objects [chapter 12]

- Stress and strain
- Elastic modulus

Today: Gravity [chapter 13]

- Newton’s laws of gravitation
- Gravitational fields and potential energy

Announcements

This week: **No problem set on Wednesday**
Practice exam posted instead

Wednesday November 9: **Midterm 2**

Gravity

Gravity is an attractive force

Classically, it is described by **Newton's laws of gravity**

Our modern understanding is captured by **general relativity**

Example: Calculate the relative correction to the acceleration due to gravity, g = 9.81 m/s^2, induced by the complete expression for Newton's law of gravitation.

Gravitational field

Acceleration due to gravity can be represented as a **field**

- mathematical quantity defined at every point in space (more generally, spacetime)

Nonlocal effect of gravity can be represented by a gravitational field

- acceleration due to gravity is a **vector field**

Multiple choice

Instructions: Consider the following question. After you have had a chance to think, I will ask you to raise your hands to indicate your answer.

Question: A test mass is located off-center in the interior of a ring of uniform mass density. What is the direction of the gravitational force on the test mass, due to the ring?

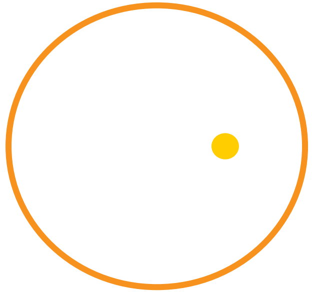
(a) leftward

(b) rightward

(c) upward

(d) downward

(e) the force is zero within the ring



Gravitational potential energy

So far our expression for potential energy due to gravity has been an approximation

- valid only close to the Earth's surface
- useful when one mass is much much larger than the other

More generally, the gravitational potential energy can be expressed as a line integral

Multiple choice

Instructions: Consider the following question. After you have had a chance to think, I will ask you to raise your hands to indicate your answer.

Question: We will have a review lecture before midterm 2 on Wednesday November 9. Would you like the review lecture on Friday November 4 or Monday November 7?

Summary

Topics

Today: Gravity [chapter 13]

- Newton's laws of gravitation
- Gravitational fields and potential energy

Wednesday: Gravity continued [chapter 13]

- Conservation of energy
- Escape velocity
- Black holes

Announcements

This week:	**No problem set on Wednesday** **Post a practice exam**
Wednesday November 9:	**Midterm 2**

PHYSICS 101 - HONORS

Lecture 31   10/31/22

Newton's Law of Gravitation (slide 3)

All mass attracts all other mass with a force proportional to the product of the masses and inversely proportional to the square of the distance between them

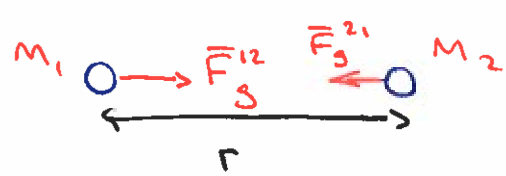
"inverse square law"

$$\vec{F}_g = \frac{G\, m_1 m_2}{r^2}\, \hat{r}_{12}$$

$$G = 6.674 \times 10^{-11}\ \frac{Nm^2}{kg^2}$$

Notes:

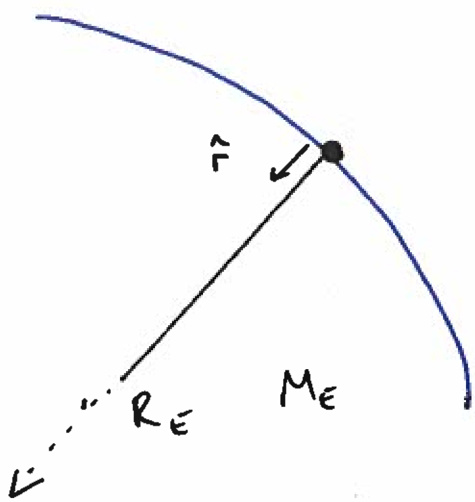
- action at a distance, no contact required!
- $1/r^2$ never goes to zero in a finite Universe!
- $F_g^{12} = F_g^{21}$ and $\vec{F}_g^{12} = -\vec{F}_g^{21}$



Acceleration example (slide 4)

Near the surface of the Earth we approximate the acceleration due to gravity as $g = 9.81\ m/s^2$

But where does this come from? Newton's law of gravitation!



At the surface of the Earth

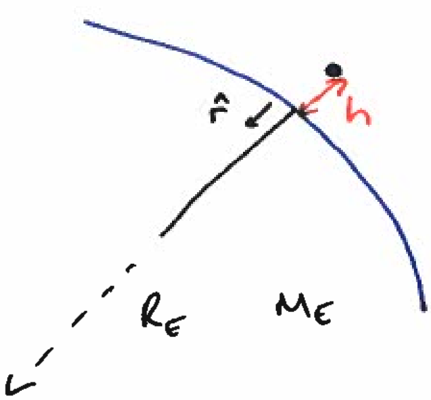
$$\bar{F}_g = \frac{G m M_E}{R_E^2}\hat{r} = m\bar{a}_g$$

$$\Rightarrow \boxed{\bar{a}_g = \frac{G M_E}{R_E^2}\hat{r}}$$

N.B. $a_g = \frac{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}{(6.371 \times 10^6)^2}$ ← mean radius

$= 9.81953$

But what if we are at the top of a building



Now $\bar{F}_g = \frac{G m M_E}{(R_E + h)^2}\hat{r} = m\bar{a}_g$

$$\bar{a}_g = \frac{G M_E}{(R_E + h)^2}\hat{r}$$

Assume that $R_E \gg h$

$$\Rightarrow \bar{a}_g = \frac{G M_E}{R_E}\left(1 + \frac{h}{R_E}\right)^{-2}\hat{r} \simeq \frac{G M_E}{R_E}\left(1 - \frac{2h}{R_E} + 3\left(\frac{h}{R_E}\right)^2 + \dots\right)\hat{r}$$

We can write this as

$$\bar{a}_g \simeq g\left(1 - \frac{2h}{R_E} + 3\left(\frac{h}{R_E}\right)^2 + \dots\right)\hat{r}$$

Estimate corrections

- tallest building on Earth is Burj Khalifa (Dubai)

$$h = 828 \Rightarrow \frac{2h}{R_E} = \frac{2 \cdot 828}{6.371 \times 10^6} = \underline{2.6 \times 10^{-4}}$$

↑ 0.025 % correction

- tallest mountain is Mt Everest

$$h = 8848\,m \Rightarrow \frac{2h}{R_E} = \frac{2 \cdot 8848}{6.371 \times 10^6} = \underline{2.8 \times 10^{-3}}$$

↑ 0.25 % correction

A 1% correction requires

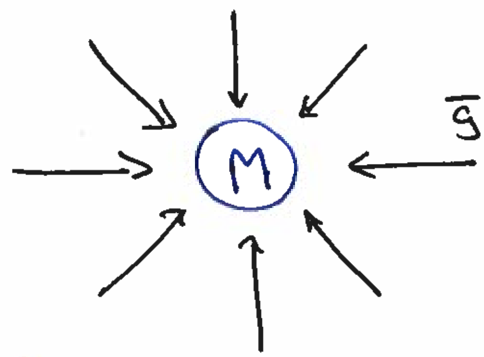
$$\frac{2h}{R_E} = 0.01 \Rightarrow h = R_E \cdot \frac{0.01}{2} = \underline{31\ km} \qquad (!)$$

↑ The Kármán line defines the boundary of Earth's atmosphere and outer space ~ 100 km

Gravitational field (slide 5)

Mass creates a vector field, a function defined at every point in space (time)

$$\bar{g} = -\frac{GM}{r^2}\hat{r}$$



For a spherical mass, the field lines are directed radially inwards

To find the force due to this field, we introduce a test particle of mass m

$$\Rightarrow \bar{F}_{test} = m\bar{g} = -m \cdot \frac{GM}{r^2}\hat{r}$$

Gravitational potential (slide 7)

$E_p = mgh$ assumes $h \ll R_E$ and $m \ll M_E$

True (more general) definition

$$\Delta U = -W_g = -\int_{\bar{r}_1}^{\bar{r}_2} \bar{F}(\bar{r}) \cdot d\bar{r}$$

Assume we bring mass along a straight line directed radially (gravity is conservative $\Rightarrow$ path does not matter!)

$$\Delta U = U_f - U_i = -\int_{\bar{r}_1}^{\bar{r}_2} \left(-\frac{GmM}{r^2}\right) \hat{r} \cdot \hat{r}\, dr$$

$$= GmM \int_{r_1}^{r_2} \frac{dr}{r^2} = GmM \left(-\frac{1}{r}\right)\Big|_{r_1}^{r_2}$$

$$= -GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

In other words

$$\boxed{U_f - U_i = -GmM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$$

Standard to take $r_i = \infty$, so $U_i = 0$ and write

$$\boxed{U = -\frac{GmM}{r}}$$