Physics 101H

General Physics 1 – Honors

Lecture 29 – 10/27/22

Static equilibrium

Summary

Topics

Yesterday: Angular motion [chapter 11]

- Examples!
- Gyroscopes

Today: Static equilibrium [chapter 12]

- Static equilibrium
- Examples

Think-pair-share

Instructions: Consider the following question. After you have had a chance to think, we will vote by show of hands and then I will ask you to discuss your answers with your neighbour, before we vote again.

Question: A ball lies on a rug, and the rug is accelerated to the right. Assume that there is at least some friction between the ball and the rug. The ball's acceleration is a (defined so rightward acceleration is positive) and the ball's angular acceleration is α (defined so clockwise is positive). Then:

(a) a is positive, and α is positive

(b) a is positive, and α is negative

(c) a is negative, and α is positive

(d) a is negative, and α is negative

(e) a is zero, and α is positive

Equilibrium

Knowing when things will not move can be important

- for example, when at the top of a ladder leaning against a wall

An object in **equilibrium** does not experience any accelerations (linear or angular):

- zero net force
- zero net torque

Static equilibrium means that the object is at rest in our chosen frame of reference

There is no real physical difference between equilibrium and static equilibrium, because the laws of physics are the same in all inertial reference frames, but sometimes it is convenient to pick a frame in which the object is at rest.

Centre of gravity

For rigid objects close to Earth, treat gravity as if it acts at the **centre of gravity**

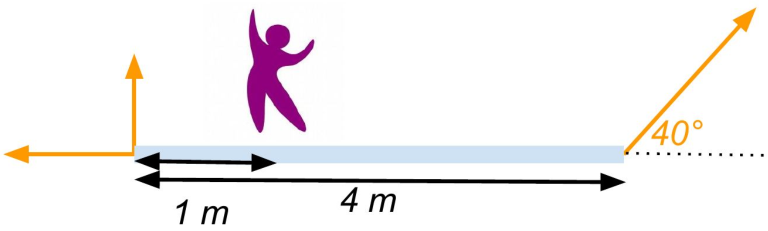
For all practical purposes, this is the same point as the centre of mass

Recall the definition of centre of mass:

- “average” position of the mass of the object
- force acting on the centre of mass causes linear motion, but not rotation

If the centre of gravity does not lie above a point of contact with the ground, gravity generates a torque that leads to **instability**

Example: A uniform plank of mass 60 kg and length 4 m is supported by three ropes. Find the tension in each rope when a person of mass 70 kg is a distance of 1 m from the left end of the plank. Assume that the plank is in equilibrium.



Example: A human arm weighs 41.5 N. Determine the magnitudes of the tension forces in the deltoid (shoulder) muscle and that exerted by the shoulder on the humerus (upper arm bone) to hold the arm straight out. Assume that the deltoid acts on the arm with an angle of 12° at a distance of 8 cm from the joint, and the centre of gravity of the arm is at a distance of 29 cm from the joint.

Summary

Topics

Today: Static equilibrium [chapter 12]

- Static equilibrium
- Examples

Tomorrow: Deformable objects [chapter 12]

- Stress and strain
- Elastic modulus

Next week: Gravity [chapter 13]

- Newton's laws
- Gravitational potential energy and escape velocity
- Kepler's laws of planetary motion

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Ball and rug question (slide 3)

(b) Friction at the bottom of the ball is directed rightwards ⇒ positive linear acceleration

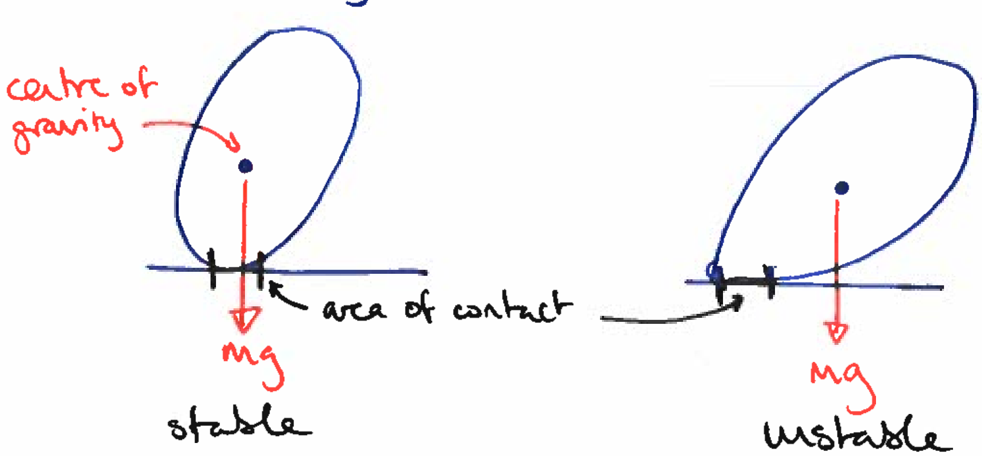
⇒ torque due to friction causes rotation in anticlockwise direction (negative)

Equilibrium and centre of gravity (slides 4/5)

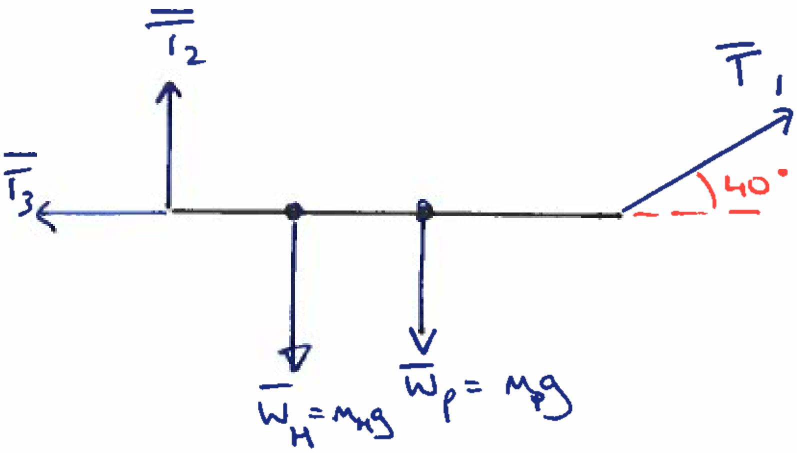
Equilibrium:

- $\bar{F}_{net} = \sum_i \bar{F}_i = 0$
- $\bar{\tau}_{net} = \sum_i \bar{\tau}_i = 0$ around any axis

Stability



Plank example (slide 6)



$M_p = 60$ kg

$M_H = 70$ kg

Equilibrium $\Rightarrow \sum_i \overline{F}_i = 0$

$\Rightarrow \overline{T}_1 + \overline{T}_2 + \overline{T}_3 + \overline{W}_H + \overline{W}_p = 0$

Horizontally: $T_1 \cos 40 - T_3 = 0 \Rightarrow T_3 = T_1 \cos 40°$ (*)

Vertically $T_1 \sin 40 + T_2 - M_H g - M_p g = 0$ (**)

Unfortunately this is two equations for three unknowns

Need to use $\sum_i \overline{\tau}_i = 0$

Consider rotation around left end

$\Rightarrow \overline{\tau}_1 + \overline{\tau}_2 + \overline{\tau}_3 + \overline{\tau}_H + \overline{\tau}_p = 0$

$\overline{\tau}_2 = \overline{\tau}_3 = 0$ because $\overline{r}_2 = \overline{r}_3 = 0$

Define anticlockwise as positive and use

$|\overline{\tau}| = |\overline{r}|\,|\overline{F}| \sin\vartheta$

$$\Rightarrow |\tau_1| = 4 T_1 \sin 40^\circ$$

$$|\tau_H| = 1 \cdot M_H g \cdot \sin 90 = M_H g$$

$$|\tau_p| = 2 \cdot m_p g \cdot \sin 90 = 2 m_p g$$

So

$$4T_1 \sin 40 - M_H g - 2m_p g = 0$$

$$\Rightarrow 4T_1 \sin 40 = (M_H + 2m_p) g$$

$$\Rightarrow T_1 = \frac{(M_H + 2m_p) g}{4 \sin 40^\circ}$$

$$= \frac{(70 + 2 \cdot 60) \cdot 9.81}{4 \sin 40}$$

$$= 724.928 \text{ N}$$

$(*) \Rightarrow$

$$T_3 = T_1 \cos 40$$

$$= 724.928 \cos 40$$

$$= 555.327 \text{ N}$$

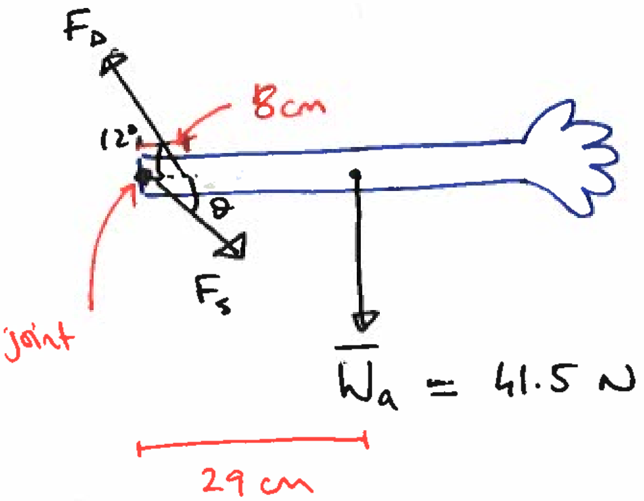
$$(**) \Rightarrow T_2 = (M_H + m_p) g - T_1 \sin 40$$

$$= (70 + 60) \cdot 9.81 - 724.928 \sin 40$$

$$= 809.325 \text{ N}$$

$T_1 = \underline{725 \text{ N}}$, $T_2 = \underline{809 \text{ N}}$, $T_3 = \underline{555 \text{ N}}$

Arm example (slide 7)



Equilibrium $\Rightarrow \sum_i \overline{F}_i = 0$

$\Rightarrow \overline{F}_D + \overline{F}_s + \overline{W}_a = 0$

Horizontally: $F_D \cos 12 = F_s \cos \theta$ (*)

Vertically: $F_D \sin 12 - F_s \sin \theta - W_a = 0$ (**)

Consider rotation around axis through joint, ↻ = +ve

$$\sum_i \overline{\tau}_i = 0 \Rightarrow \overline{\tau}_D + \overline{\tau}_a = 0$$

$$0.08 \cdot F_D \sin 12 - 0.29 \cdot W_a \sin 90 = 0$$

$$\Rightarrow 0.08\, F_D \sin 12 = 0.29\, W_a$$

$$\Rightarrow F_D = \frac{0.29 \cdot 41.5}{0.08 \sin 12} = 723.564\ N$$

$F_D = 724\ N$

(**) $\Rightarrow F_s \sin \theta = -W_a + F_D \sin 12$

combine with (*) by dividing

$$\tan \theta = \frac{F_D \sin 12^\circ - W_a}{F_D \cos 12} \Rightarrow \theta = \arctan\left(\frac{723.564 \sin 12 - 41.5}{723.564 \cos 12}\right)$$

$$= 8.75^\circ$$

$$\Rightarrow F_s = \frac{F_D \cos 12^\circ}{\cos \theta} = \frac{723.564 \cos 12}{\cos 8.75} = 716\ N$$