Physics 101H

General Physics 1 – Honors

Lecture 28 – 10/26/22

Rotational motion and gyroscopes

Summary

Topics

Monday: Angular motion [chapter 11]

- Rolling motion
- Angular momentum
- Rotational dynamics

Today: Angular motion [chapter 11]

- Examples!
- Gyroscopes

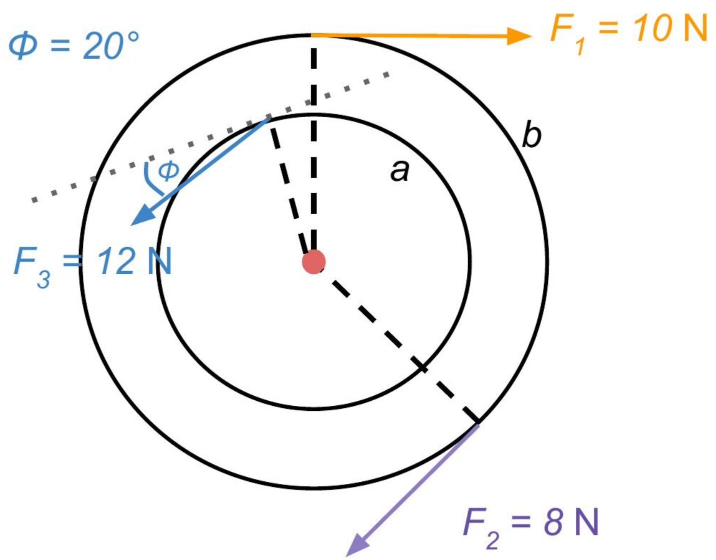
Announcements

Today: **Problem Set 5 due today**
Problem Set 6 posted today

Quick quiz

Instructions: This quiz is for your own learning. There are three questions and each question has two columns. Write your own solution, without reference to your notes, the textbook, or your neighbour, **in the first column**. Once you have tried to answer all the questions, discuss the questions with a neighbour and fill in any incomplete answers **in the second column**. Keep your sheet for future reference.

Example: Find the net torque on the wheel, about the axle, if a = 7 cm and b = 21 cm. Assume that anticlockwise rotations are positive.



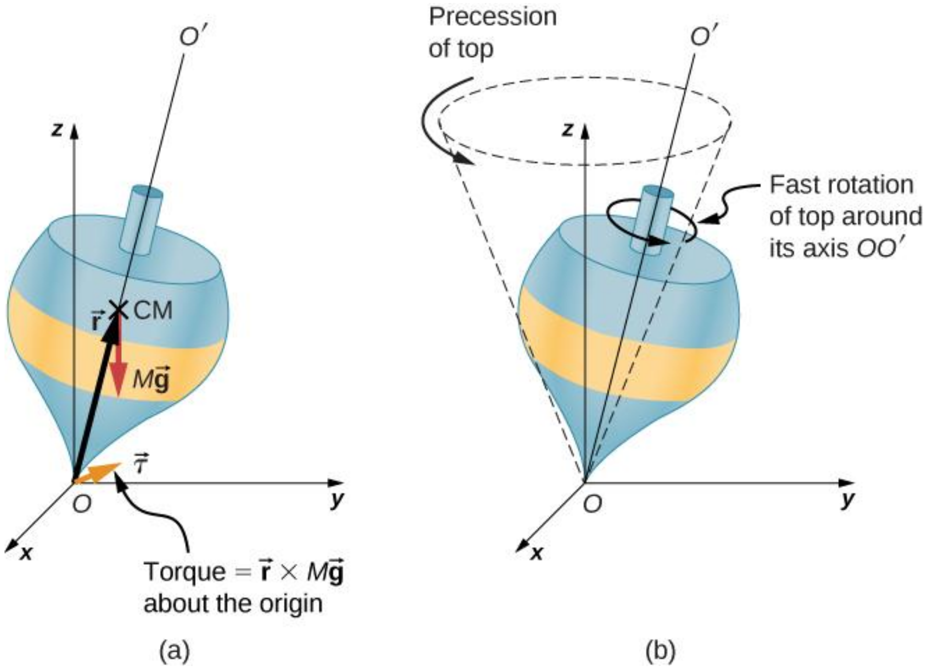
Example: Rigid rods of negligible mass connect three particles of mass 4 kg, 2 kg, and 3 kg at distances of 3 m, –2 m and –4 m from the x axis, respectively. The system rotates about the x axis with an angular speed of 1.1 rad/s. Find (a) the moment of inertia about the x axis; (b) the rotational kinetic energy of the system; (c) the tangential speed of each particle; and (d) the total translational kinetic energy of the system.

Gyroscope

A **gyroscope** is a spinning object for which the axis of rotation is free to move.

Gyroscopes preserve the orientation of their axis when they are spinning and can be used to detect rotation – this is super useful in, for example, space and is used for navigation in autonomous vehicles and robots

Gyroscopes undergo **precession**



Two minute essay

Instructions: Draw a diagram for the following topic. You have two minutes. You may not use your notes and you should not consult with others around you. Your answer will not be graded; your answer is for your own learning.

Question: Recall your two-column table from Lecture 26 (one column for linear/translational motion and one for rotational motion). Update this table to include all the relevant quantities, kinematic equations and dynamics equations that we have discussed for rotational motion so far. Once you have finished, compare your table with your neighbours.

Want more practice?

Try the following problems **Chapter 10** of the textbook:

- Conceptual questions: 1, 3, 5, 9, 13, 17, 19, 21, 23
- Rotational variables and kinematics: 29, 33, 39, 43, 49, 53, **121**
- Moment of inertia: 59, 63, 65, 69, **123**
- Torque: 71, 75, 77, 81
- Newton's second law for rotations: 85, 89, 95

Answers are provided for questions with **blue** numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

Want more practice?

Try the following problems **Chapter 11** of the textbook:

- Conceptual questions: 1, 3, 5, 9, 11, 15, 17
- Rolling motion: 23, 27, 29, 33, **83**
- Angular momentum: 35, 37, 43, 45, 51, **85**, **97**
- Angular momentum conservation: 55, 57, 61, 65, 71, **81**
- Precession: 77

Answers are provided for questions with **blue** numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

Summary

Topics

Today: Angular motion [chapter 11]

- Examples!
- Gyroscopes

Tomorrow: Statics [chapter 12]

- Static equilibrium
- Deforming objects

Announcements

Today: **Problem Set 5 due today**
Problem Set 6 posted today

PHYSICS 101 - HONORS

Lecture 28 10/26/22

Torque example (slide 4)

Recall $\bar{\tau} = \bar{r} \times \bar{F}$

First let's identify whether torques are positive or negative, and then we can use

$$|\bar{\tau}| = |\bar{r}||\bar{F}| \sin\theta$$

F_1 ↷ => $-ve$

F_2 ↷ => $-ve$

F_3 ↶ => $+ve$

$$\bar{\tau}_{net} = \bar{\tau}_1 + \bar{\tau}_2 + \bar{\tau}_3$$

$$\tau_{net} = -b F_1 \sin 90^\circ - b F_2 \sin 90^\circ + a F_3 \sin(90-20)$$

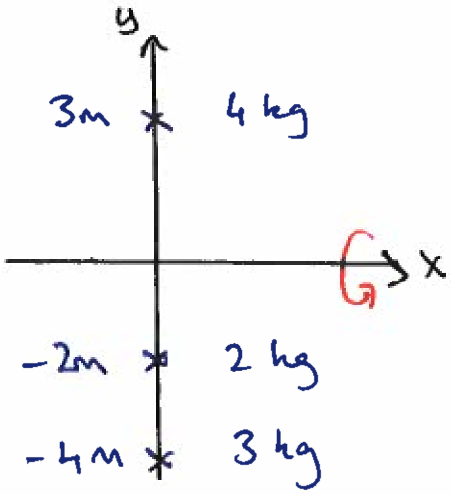
$$= -b(F_1 + F_2) + a F_3 \sin 70^\circ$$

$$= -0.21(10+8) + 0.07 \cdot 12\,N \cdot \sin 70^\circ$$

$$= -3.0 \ Nm$$

↑ clockwise!

Three mass example (slide 5)



(a) $I = \sum_{i=1}^{3} m_i r_i^2$

$= 4\cdot 3^2 + 2\cdot(-2)^2 + 3\cdot(-4$

$= \underline{92 \text{ kg m}^2}$

(b) $E_K = \frac{1}{2} I \omega^2$

$= \frac{1}{2}\cdot 92 \cdot 1.1^2$

$= \underline{55.7 \text{ J}}$

(c) $v = \omega r$

$\Rightarrow v_1 = \omega y_1 = 1.1 \cdot 3 = \underline{3.3 \text{ m/s}}$

$v_2 = \omega y_2 = 1.1 \cdot 2 = \underline{2.2 \text{ m/s}}$

$v_3 = \omega y_3 = 1.1 \cdot 4 = \underline{4.4 \text{ m/s}}$

(d) $E_K = E_K^1 + E_K^2 + E_K^3$

$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$

$= \frac{1}{2}\cdot 4 \cdot 3.3^2 + \frac{1}{2}\cdot 2 \cdot 2.2^2 + \frac{1}{2}\cdot 3 \cdot 4.4^2$

$= \underline{55.7 \text{ J}}$ (!)

Q: Why should this not be a surprise?

Gyroscopes (slide 6)

In the absence of rotational motion, gravity causes a top to fall over, by applying a torque about the pivot point.

When spinning, there is still a torque

$\bar{\tau} = \bar{r} \times \bar{F}$

$|\bar{\tau}| = |\bar{r}||\bar{F}| \sin\theta$

$= rmg \sin\theta$ ← points ⊥ to $\bar{r}$ and $\bar{F}$

eg if $\bar{F}_g \propto \hat{z}$ and $\bar{r}$ is in y-z plane, then $\bar{\tau} \propto \hat{x}$

This torque causes angular momentum to change because $\bar{\tau} = \frac{d\bar{L}}{dt}$ ← $\bar{L}$ is ∥ to $\bar{r}$

Since $\bar{\tau}$ is ⊥ to $\bar{L}$, it only changes the direction of $\bar{L}$, not $|\bar{L}|$

↑ axis of rotation rotates around z axis ← <u>precession</u>

Angular speed of precession is

$$\omega_p = \frac{mgr}{I\omega}$$

← proof is in section 11.4 of textbook.

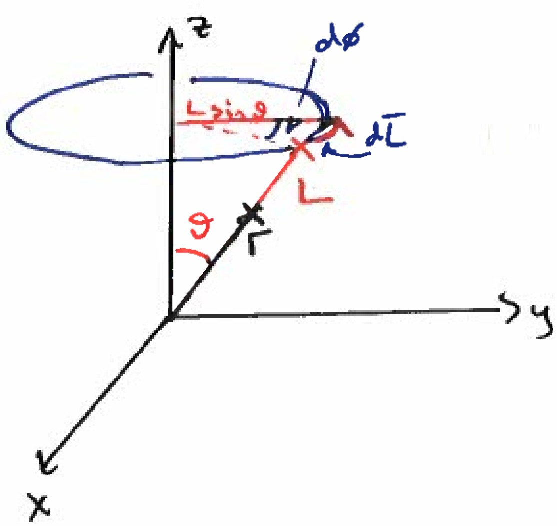
[For completeness

$$\frac{dL}{dt} = rMg\sin\theta \quad \Rightarrow \quad dL = rMg\sin\theta\, dt$$

but $$d\phi = \frac{dL}{L\sin\theta} = \frac{rMg\sin\theta\, dt}{L\sin\theta} = \frac{rMg}{L}dt$$

$$\Rightarrow \omega = \frac{d\phi}{dt} = \frac{rMg}{L} = \frac{rMg}{I\omega}$$

Note



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Translational and rotational motion

Translational

$\bar{r}$

$\bar{v} = \dfrac{d\bar{r}}{dt}$

$\bar{a} = \dfrac{d\bar{v}}{dt} = \dfrac{d^2\bar{r}}{dt^2}$

$$\left\{ \begin{array}{l} s = r\vartheta \\ v_t = \omega r \\ \alpha = \dfrac{a_t}{r} \\ a_c = \dfrac{v_t^2}{r} \end{array} \right\}$$

$\bar{v} = \bar{a}t + \bar{v}_0$

$\bar{x} = \dfrac{\bar{a}t^2}{2} + \bar{v}_0 t + x_0$

M

$E_K^{rot} = \frac{1}{2} I \omega^2$

$\bar{L} = \bar{r} \times \bar{p} = I\bar{\omega}$

$\bar{\tau} = \dfrac{d\bar{L}}{dt} = \bar{r} \times \bar{F}$

$\bar{\tau}_{net} = I\bar{\alpha}$

Rotational

$\bar{\vartheta}$

$\bar{\omega} = \dfrac{d\bar{\vartheta}}{dt}$

$\bar{\alpha} = \dfrac{d\bar{\omega}}{dt} = \dfrac{d^2\vartheta}{dt^2}$

$\bar{\omega} = \bar{\alpha}t + \bar{\omega}_0$

$\bar{\vartheta} = \dfrac{\bar{\alpha}t^2}{2} + \bar{\omega}_0 t + \bar{\vartheta}_0$

$I = \int r^2 \, dm$ or $\sum_i m_i r_i^2$

$E_K^{lin} = \frac{1}{2} M v^2$

$\bar{p} = M\bar{v}$

$\bar{F} = \dfrac{d\bar{p}}{dt}$

$\bar{F}_{net} = M\bar{a}$