Physics 101H

General Physics 1 – Honors

Lecture 25 – 10/20/22

Angular motion

Summary

Topics

Yesterday: Rockets [chapter 9]

- Rockets
- Propulsion and conservation of momentum
- One more centre of mass example

Today: Angular motion [chapter 10]

- Rigid bodies
- Angular motion
- Torque
- Moment of inertia

Rigid bodies

All motion so far has been for point particles

- Even centre of mass motion was treated as point particle motion

No we will consider **rigid**, **extended objects**

- Rigid = nondeformable (relative positions of internal parts do not change)
- Extended = cannot be treated as a point particle

Extended objects **can rotate**

Torque and moment of inertia

When a force is exerted on a rigid object that is allowed to rotate, the object tends to rotate about that axis

Torque

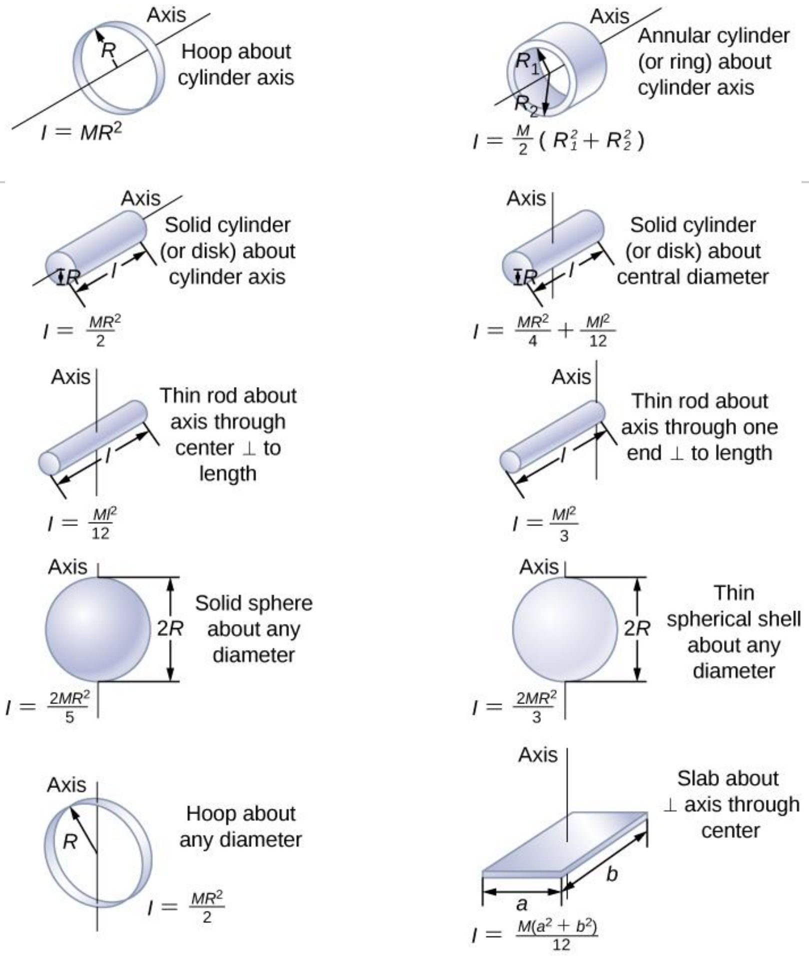
- measures the “turning power” of an applied force

Moment of inertia

- generalises our idea of mass as the resistance to linear motion
- measures the resistance to rotational motion

Common moments of inertia

Like Taylor series, we can look up common moments of inertia for simple geometrical bodies



Summary

Topics

Today: Angular motion [chapter 10]

- Rigid bodies
- Angular motion
- Torque
- Moment of inertia

Tomorrow: Moment of inertia [chapter 10]

- Moment of inertia calculations
- Parallel axis theorem
- Rotational kinetic energy

Summary

Topics

Today: Rockets [chapter 9]

- Rockets
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Tomorrow: Angular motion [chapter 10]

- Angular motion
- Moment of inertia
- Torque

Announcements

This week: **Problem Set 5 posted today**

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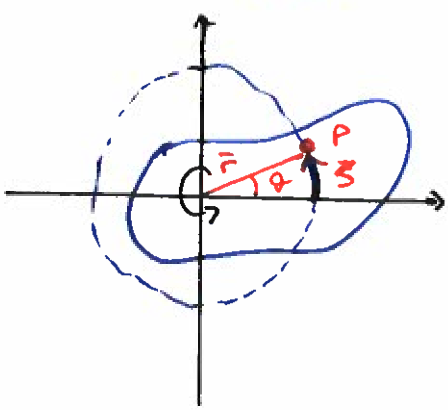
Rigid bodies (slide 3)

Translational	Rotational
displacement $\bar{r}$	angular displacement $\bar{\theta}$
velocity $\bar{v} = \frac{d\bar{r}}{dt}$	angular velocity $\bar{\omega} = \frac{d\bar{\theta}}{dt}$
acceleration $\bar{a} = \frac{d\bar{v}}{dt}$	angular acceleration $\bar{\alpha} = \frac{d\bar{\omega}}{dt}$

↗ vectors point along the axis of rotation, with direction given by right hand rule

Consider motion of a rigid object around some axis



point p rotates $s = r\theta$ ← radians

with linear speed $v = \frac{ds}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt} = r\omega$

tangential acceleration $a_t = r\frac{d\omega}{dt} = r\alpha$

Note the linear acceleration is $\bar{a} = \bar{a}_t + \bar{a}_r$

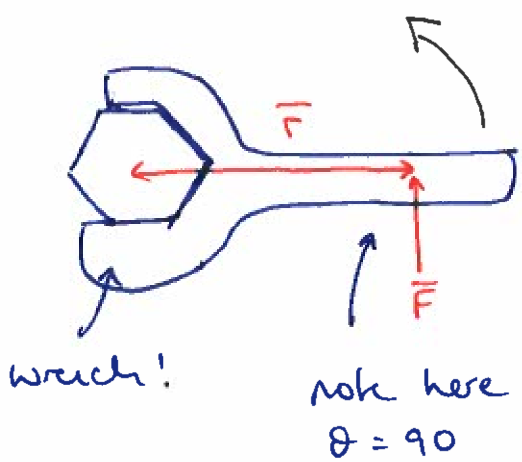
with $|\bar{a}| = \sqrt{|\bar{a}_t|^2 + |\bar{a}_r|^2} = \sqrt{r^2\alpha^2 + (v^2/r)^2} = r\sqrt{\omega^4 + \alpha^2}$

Kinematic equations for constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \Rightarrow \boxed{\omega = \alpha t + \omega_0} \quad \text{cf} \quad v = a_0 t + v_0$$

$$\omega = \frac{d\theta}{dt} \Rightarrow \boxed{\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0} \quad \text{cf} \quad x = \frac{at^2}{2} + v_0 t + x_0$$

Torque and moment of inertia (slide 4)



Torque is defined as

$$\boxed{\bar{\tau} = \bar{r} \times \bar{F}}$$

or $|\bar{\tau}| = |\bar{r}|\,|\bar{F}| \sin\theta$

Note torque is larger when the force is further away from the axis of rotation ↖

It also depends on the angle between $\bar{F}$ and $\bar{r}$ – if $\bar{F} \propto \bar{r}$ then $\bar{\tau} = 0$.

think about opening a door ↓

This tangential force causes a tangential acceleration

$$F_t = m a_t \Rightarrow \tau = F_t r = m a_t r = r(\alpha r) r = \underbrace{m r^2}\alpha$$

called the moment of inertia

Notice similarities

translational	rotational
$F = ma$	$\tau = I\alpha$

Moment of inertia measures resistance to rotational motion !

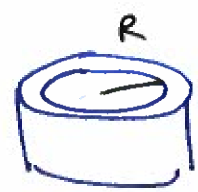
Moment of inertia :

← always measured with respect to some axis

- single particle $I = mr^2$
- system of particles $I = \sum_{i=1}^{N} m_i r_i^2$
- extended (continuous object) $I = \int r^2 \, dm$

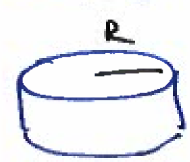
Useful results

- thin cylindrical shell



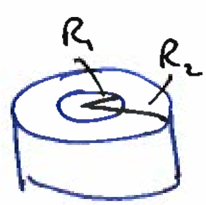
$I = MR^2$

- solid cylinder



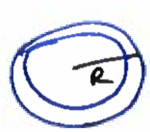
$I = \frac{1}{2}MR^2$

- hollow cylinder



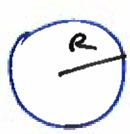
$I = \frac{1}{2} M(R_1^2 + R_2^2)$

- spherical shell



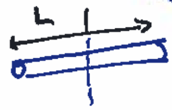
$I = \frac{2}{3}MR^2$

- sphere



$I = \frac{2}{5}MR^2$

- rod



$I = \frac{1}{12}ML^2$