Physics 101H

General Physics 1 – Honors

Lecture 24 – 10/19/22

Rockets

Summary

Topics

Monday: Centre of mass [chapter 9]
- Centre of mass examples

Today: Rockets [chapter 9]
- Rockets
- Propulsion and conservation of momentum
- One more centre of mass example

Tomorrow: Angular motion [chapter 10]
- Rigid bodies
- Angular motion
- Torque
- Moment of inertia

Announcements

This week: **Problem Set 5 posted today**

Two minute essay

Instructions: Write one paragraph on the following topic. You have two minutes. You may not use your notes and you should not consult with others around you. Your answer will not be graded; your answer is for your own learning and you don't need to share your answer.*

Question: Describe the role of conservation laws in collisions in one and two dimensions. You should include both elastic and inelastic collisions.

Rockets

Rockets are propelled by expelling a fuel at high velocity

Question: How does conservation of momentum explain how rockets work?

Example:

82. Unreasonable Results Squids have been reported to jump from the ocean and travel 30.0 m (measured horizontally) before re-entering the water.

(a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0°, assuming negligible lift from the air and negligible air resistance.

(b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at 12.0 m/s; gravitational force and friction are neglected.

(c) What is unreasonable about the results?

(d) Which premise is unreasonable, or which premises are inconsistent?

Example: Find the y position of the centre of mass of an isosceles triangle, of density ρ, that is h high, $2w$ wide, and t thick (a three-dimensional triangle?).

Want more practice?

Try the following problems **Chapter 9** of the textbook:

- Conceptual questions: 1, 3, 9, 13, 17
- Momentum: 19, 23, **88**
- Impulse: 27, 31, 33, **92**
- Momentum conservation: 37, 39, 41
- Collisions: 45, 49, 51, 55, 59, 61, **96**, **106**
- Centre of mass: 63, 65, **69**, **71**
- Rockets: 77, 81, **112**

Answers are provided for questions with blue numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

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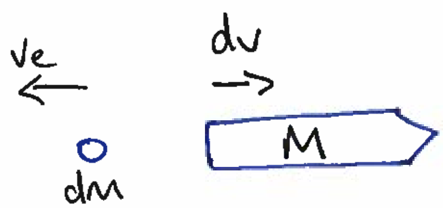
PHYSICS 101 - HONORS

Lecture 24

Rockets (slide 4)

Rockets eject fuel, which has momentum, and the rocket must have momentum in the opposite direction, to ensure momentum is conserved



conservation of momentum $\Rightarrow$ $v_e \, dm = M dv$

But as mass is expelled, M must change!

$$dM = -dm \Rightarrow -v_e \, dM = M dv$$

$$\Rightarrow -v_e \frac{dM}{M} = dv$$

$$\Rightarrow -v_e \int_{M_i}^{M_f} \frac{dM}{M} = \int_{v_i}^{v_f} dv$$

$$-v_e \ln M \Big|_{M_i}^{M_f} = v \Big|_{v_i}^{v_f}$$

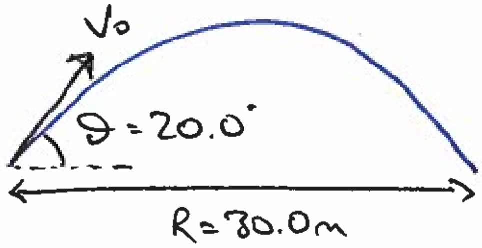
$$\Rightarrow v_f - v_i = -v_e (\ln M_f - \ln M_i)$$

or $$\Delta v = v_e (\ln M_i - \ln M_f)$$

$$\boxed{\Delta v = v_e \ln \frac{M_i}{M_f}}$$ $\leftarrow$ the "rocket equation"

Squid example (slide 5)

(a)



Easiest to use the range equation

$$R = \frac{2v_0^2}{g} \sin\vartheta \cos\vartheta$$

$$\Rightarrow v_0 = \sqrt{\frac{gR}{2\sin\vartheta\cos\vartheta}} = 30.2605 \text{ m/s}$$

↙ keep sig figs for next part

$$\boxed{v_0 = 30.3 \text{ m/s}}$$

(b) Recall

$$\Delta v = v_e \ln\frac{M_i}{M_f} \Rightarrow \ln\frac{M_i}{M_f} = \frac{\Delta v}{v_e}$$

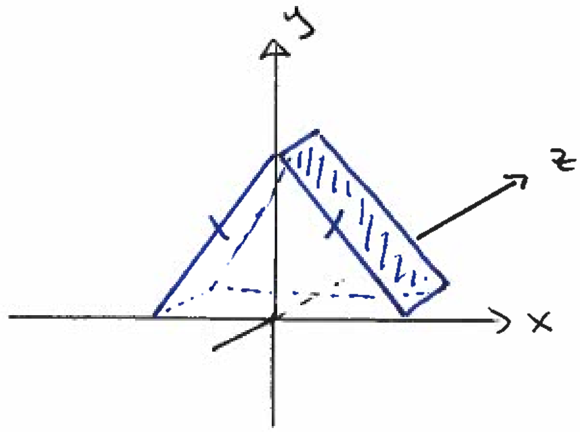
$$\Rightarrow \frac{M_i}{M_f} = \exp\left(\frac{\Delta v}{v_e}\right)$$

More natural to express this as

$$\frac{M_f}{M_i} = \frac{1}{\exp\left(\frac{\Delta v}{v_e}\right)} = e^{-\Delta v/v_e} = \exp\left(-\frac{30.2605}{12.0}\right)$$

$$= 0.080$$ ⇒ it would have to eject $\boxed{92\%}$ of its mass!

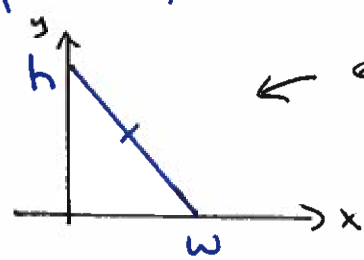
Triangle example (slide 6)



Symmetry tells us x-component must lie along line from base to point, which we define to be the y-axis

The z-component must be $t/2$.

For the y component, consider just one half of the triangle



← equation of line $y = mx + b$

in this case $y = -\frac{h}{w}x + h$

other half: $y = \frac{h}{w}x - h$

$$y_{cm} = \frac{1}{M}\int y\,dm$$

↖ what is dm?

Density is $\rho = \frac{M}{V} \Rightarrow \rho = \frac{dm}{dV}$

$\Rightarrow dm = \rho\, dV$

$= \rho\, dx\,dy\,dz$

$$\Rightarrow y_{cm} = \frac{2}{M}\iiint y\rho\, dx\,dy\,dz$$

But what are the limits of integration?

$z \in [0, t]$ independent of x, y

x range depends on y

$y \in [0, h]$, once we take care of x

At a given y value, we integrate from

0 to $-\frac{h}{w}x + h$ $\quad x \geq 0 \quad \Rightarrow x = -\frac{w}{h}(y-h)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = \frac{w}{h}(h-y)$

$\frac{h}{w}x + h$ to 0 $\quad x < 0 \quad \Rightarrow x = \frac{w}{h}(y-h)$

Thus

$$y_{cm} = \frac{1}{M}\int_0^t dz \int_0^h \left[\int_{\frac{w}{h}(y-h)}^{0} + \int_0^{\frac{w}{h}(h-y)}\right] y\rho \, dx\,dy\,dz$$

$$= \frac{\rho}{M}\int_0^t dz \int_0^h y \left[\int_{\frac{w}{h}(y-h)}^{0} dx + \int_0^{\frac{w}{h}(h-y)} dx\right] dy$$

$$= \frac{\rho}{M}\int_0^t dz \int_0^h y \int_{\frac{w}{h}(y-h)}^{\frac{w}{h}(h-y)} dx\, dy$$

$$= \frac{\rho}{M}\cdot t \cdot \int_0^h y \left[x \Big|_{\frac{w}{h}(y-h)}^{\frac{w}{h}(h-y)}\right] dy$$

$$= \frac{\rho t}{M}\int_0^h y\left(\frac{w}{h}(h-y) - \frac{w}{h}(y-h)\right) dy$$

$$= \frac{\rho t 2w}{M\, h}\int_0^h y(h-y)\, dy$$

$$= \frac{2\rho t w}{M h}\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$= \frac{2\rho t w}{M h} h \cdot h^2 \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{2\rho t w h^2}{M 6}$$

Note $\rho = \frac{M}{V} = \frac{M}{2\cdot\left(\frac{1}{2}wh\right)t} = \frac{M}{wht} \Rightarrow y_{cm} = \frac{M}{wht}\cdot\frac{twh^2}{3M}$

$$= \boxed{\frac{h}{3}}$$