Physics 101H

General Physics 1 – Honors

Lecture 23 – 10/17/22

Centre of mass

Summary

Topics

Last week:

- Collisions in two dimensions [chapter 9]
- Centre of mass [chapter 9]

Today: Centre of mass

- 2D collision example
- Centre of mass examples [chapter 9]

Announcements

This/next week: **Problem Set 5 posted on Wed Oct 19**

Example: A snooker ball moving at 5 m/s strikes a stationary snooker ball of the same mass. Afterwards, the first ball moves at 4.33 m/s at an angle of 30 degrees with respect to the original line of motion. Assuming an elastic collision, and ignoring friction and rotational motion, find the struck ball's final velocity.

Reminder: centre of mass

Centre of mass is a special point in a system

If all the mass of a system is at the centre of mass, then the translational motion of the system is unchanged

- System moves as if any net force were applied to a particle of total mass of the system located at that point
- Centre of mass is approximately the "average position" of the system's mass

Example: Find the centre of mass of a rod of length l and mass M.

Example: A projectile of mass 5 kg explodes into two fragments at some point in its trajectory. One fragment of mass 2 kg falls at a point $2R/3$, where R is the range of the projectile. Where does the other fragment fall, assuming no mass is lost in the process?

Summary

Topics

Today: Centre of mass [chapter 9]

- 2D collision example
- Centre of mass examples

Wednesday: Rockets [chapter 9]

- Rockets
- Propulsion and conservation of momentum
- One more centre of mass example

Announcements

This/next week: **Problem Set 5 posted on Wed Oct 19**

PHYSICS 101 - HONORS

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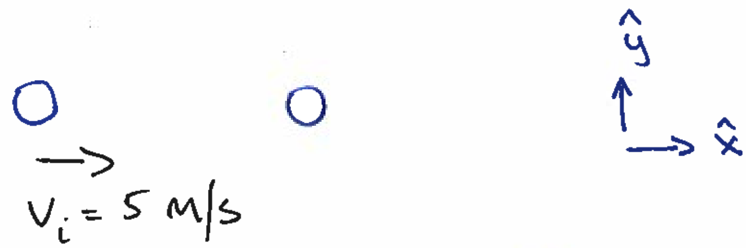
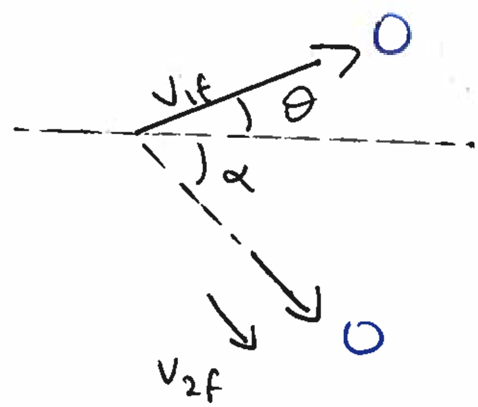
Snooker example in 2D (slide 3)

initial :



final :



$v_{1f} = 4.33$ m/s
$\theta = 30°$

$\alpha = ?$
$v_{2f} = ?$

Elastic ⇒ momentum conserved
kinetic energy conserved

Initial : $\bar{p}_i = m\bar{v}_i = m v_i \hat{x}$

Final : $\bar{p}_f = m\bar{v}_{1f} + m\bar{v}_{2f} = m(v_{1f}\cos\theta\,\hat{x} + v_{1f}\sin\theta\,\hat{y})$
$+ m(v_{2f}\cos\alpha\,\hat{x} + v_{2f}\sin\alpha\,\hat{y})$

conservation of momentum in $\hat{x}$ direction

$$\Rightarrow m v_i = m v_{1f}\cos\theta + m v_{2f}\cos\alpha$$

$$\underbrace{v_i - v_{1f}\cos\theta}_{\text{known}} = \underbrace{v_{2f}\cos\alpha}_{\text{unknown}} \quad (1)$$

Conservation of momentum in $\hat{y}$ direction

$$0 = m v_{1f} \sin\theta + m V_{2f} \sin\alpha$$

$$\Rightarrow \quad -V_{1f} \sin\theta = V_{2f} \sin\alpha \qquad (2)$$

Now divide (2) by (1)

$$\frac{V_{2f} \sin\alpha}{V_{2f} \cos\alpha} = \frac{-V_{1f} \sin\theta}{V_i - V_{1f} \cos\theta}$$

$$\Rightarrow \quad \tan\alpha = \frac{-V_{1f} \sin\theta}{V_i - V_{1f} \cos\theta}$$

$$\alpha = \arctan\left(\frac{-4.33 \sin 30^\circ}{5 - 4.33 \cos 30^\circ}\right) = -60^\circ$$

Thus

$$V_{2f} = -\frac{V_{1f} \sin\theta}{\sin\alpha}$$

$$= -\frac{4.33 \sin 30}{\sin(-60^\circ)}$$

$$= 2.5 \text{ m/s}$$

Final velocity is 2.5 m/s at angle of 60° below the original direction of the first ball.

Rod example (slide 5)

Let's orient our rod along the x-axis

This is a continuous distribution of matter/mass, so

$$\bar{r}_{cm} = \frac{1}{M}\int \bar{r}\, dm$$

In this case, the rod has linear mass density $\lambda = \frac{M}{\ell}$ $\Rightarrow$ a small volume has mass $dm = \lambda\, dx = \frac{M}{\ell} dx$

The position of the volume dm is just x

$$\Rightarrow r_{cm} = \frac{1}{M}\int x\, dm$$

$$= \frac{1}{M}\int x \frac{M}{\ell} dx$$

$$= \frac{1}{M}\cdot\frac{M}{\ell}\int_0^{\ell} x\, dx$$

$$= \frac{1}{\ell}\cdot\frac{x^2}{2}\Big|_0^{\ell}$$

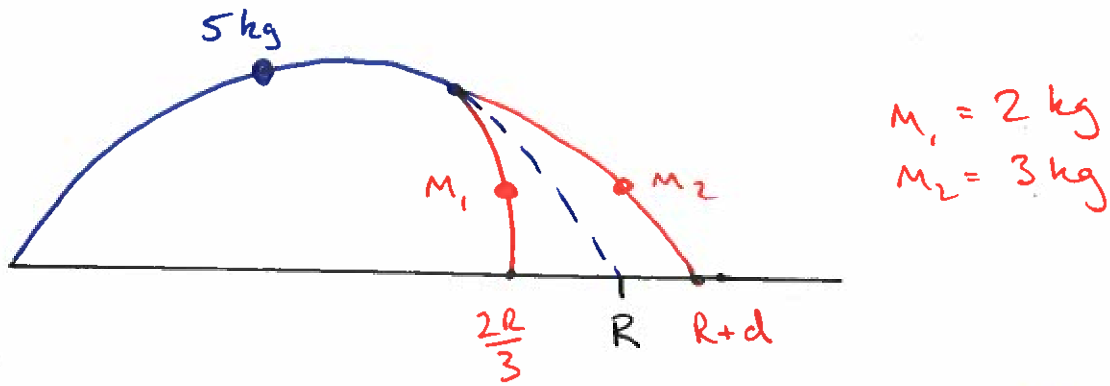
$$= \frac{1}{\ell}\cdot\frac{\ell^2}{2} = \boxed{\frac{\ell}{2}}$$

← halfway along the rod! (as we may have expected)

Projectile example (slide 6)

The key to solving this is to recognise the centre of mass continues its trajectory, even after the projectile has split!

This means the centre of mass must be at R when the two fragments land.



$$\Rightarrow \frac{1}{M}\cdot\left(M_1 \frac{2R}{3} + M_2\cdot(R+d)\right) = R \qquad M = M_1 + M_2$$

$$\Rightarrow \frac{2M_1R}{3} + M_2R + dM_2 = MR$$

$$d = \frac{MR - \frac{2}{3}M_1R - M_2R}{M_2}$$

$$= \frac{\left(M - \frac{2}{3}M_1 - M_2\right)R}{M_2}$$

$$= \frac{5 - \frac{2}{3}\cdot 2 - 3}{3}R$$

$$= \frac{2}{9}R$$

$\Rightarrow$ other projectile lands at $\boxed{\frac{11}{9}R}$