Physics 101H

General Physics 1 – Honors

Lecture 22 – 10/12/22

Centre of mass

Summary

Topics

Monday: Collisions [chapter 9]

- Elastic collisions
- Inelastic collisions
- Collisions in two dimensions

Today: Centre of mass [chapter 9]

- Centre of mass
- Calculating the centre of mass
- Centre of mass motion

Announcements

This week: **No class on Thursday or Friday**

No homework this week!

Example: Find the final velocity of a tennis ball when a tennis ball and basketball are dropped simultaneously from a height h? Assume that the tennis ball is vertically above the basketball and has a mass one tenth of the basketball, that both balls are initially stationary, and that all collisions are perfectly elastic.

Example: A pellet of mass m is fired into a block of wood of mass M, suspended from a wire. The pellet embeds itself in the block, and the combined system swings to a height h. What was the initial speed of the pellet (in terms of m, M, and h)?

Multiple choice

Instructions: Consider the following question. After you have had a chance to think, I will ask you to raise your hands to indicate your answer.

Question: How should you build a car to reduce the likelihood of injury in a head-on crash?

(a) Make the front bumper rigid;

(b) Make the front of the car crumple when large forces are applied;

(c) Make the front of the car crumple easily when small forces are applied;

(d) Make the entire front of the car rigid, so that it does not crumple at all;

(e) Install a set of springs in the front of the car so that it bounces backwards after the collision.

Collisions in two dimensions

Remember that momentum is a vector!

We can treat each component of a vector separately in an orthonormal basis.

In a two dimensional collision:

Each component of the momentum is conserved independently

Centre of mass

Why everything is a point particle

Centre of mass

Centre of mass is a special point in a system

If all the mass of a system is at the centre of mass, then the translational motion of the system is unchanged

- System moves as if any net force were applied to a particle of total mass of the system located at that point
- Centre of mass is approximately the “average position” of the system’s mass

Calculating the centre of mass

Consider:

- system of point-like discrete particles
- continuous distribution of mass – an **extended object**

Remember – the system moves as though any net force were applied to a single point-like particle of equivalent mass at the centre of mass

Spot the typo?

9.6 Center of Mass

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You may sometimes hear someone describe an explosion by saying something like, "the fragments of the exploded object always move in a way that makes sure that the center of mass continues to move on its original trajectory." This makes it sound as if the process is somewhat magical: how can it be that, in *every* explosion, it *always* works out that the fragments move in just the right way so that the center of mass' motion is unchanged? Phrased this way, it would be hard to believe no explosion ever does anything differently.

The explanation of this apparently astonishing coincidence is: We defined the center of mass precisely so this is exactly what we would get. Recall that first we defined the momentum of the system:

$$\vec{\mathbf{p}}_{\text{CM}} = \sum_{j=1}^{N} \frac{d\vec{\mathbf{p}}_j}{dt}.$$

We then concluded that the net external force on the system (if any) changed this momentum:

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}_{\text{CM}}}{dt}$$

and then—and here's the point—we defined an acceleration that would obey Newton's second law. That is, we demanded that we should be able to write

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{M}$$

which requires that

$$\vec{\mathbf{a}} = \frac{d^2}{dt^2}\left(\frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{r}}_j\right).$$

where the quantity inside the parentheses is the center of mass of our system. So, it's not astonishing that the center of mass obeys Newton's second law; we defined it so that it would.

Summary

Topics

Today: Centre of mass [chapter 9]

- Centre of mass
- Calculating the centre of mass
- Centre of mass motion

Next week:

- Centre of mass [chapter 9]
- Rockets [chapter 9]
- Angular motion [chapter 10]

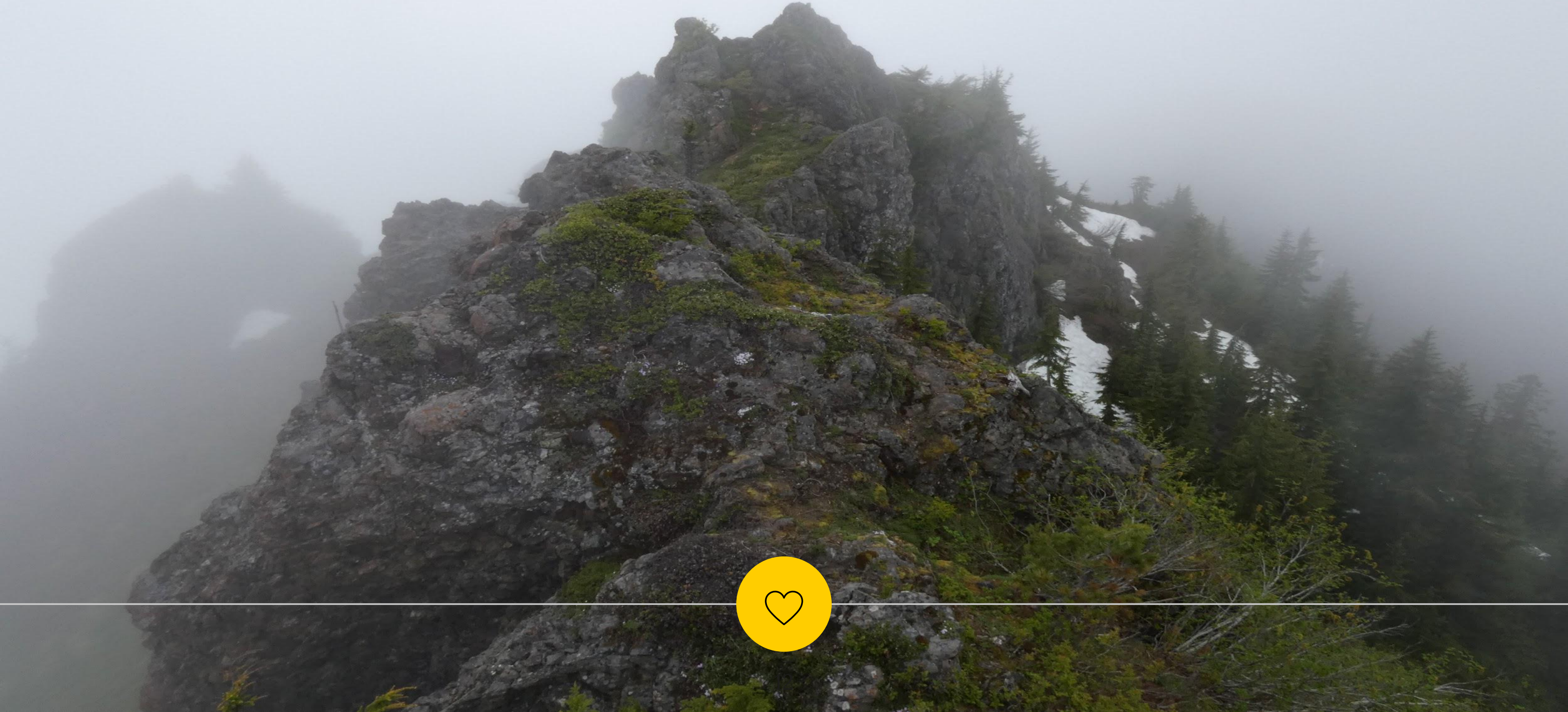
Announcements

This week: **No class on Thursday or Friday**

No homework this week!

Next week: **Pre-recorded lecture for Monday**

THIS WEEK AND NEXT WEEK: THERE ARE NO CLASSES ON THURSDAY OCTOBER 13 AND FRIDAY OCTOBER 14 MONDAY OCTOBER 17 IS PRE-RECORDED

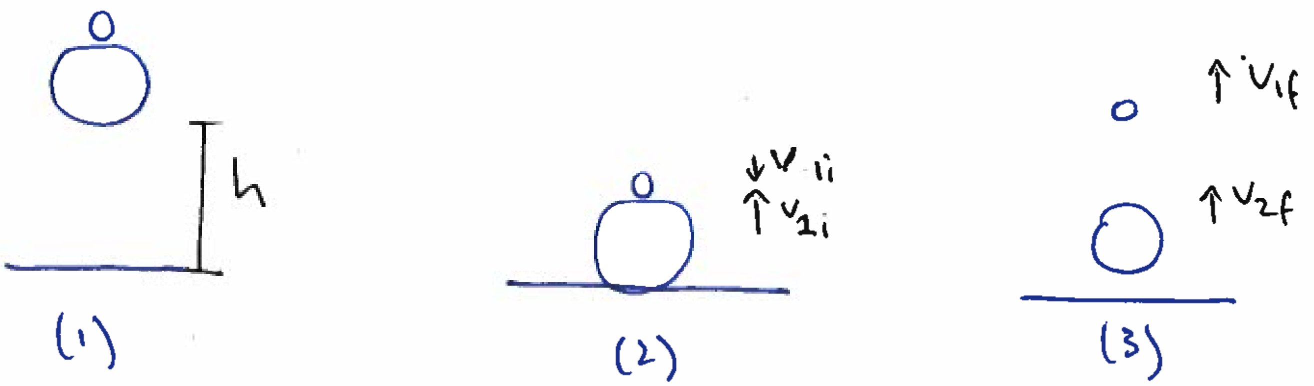


PHYSICS 101 - HONORS

Lecture 22 10/12/22

Our starting point is the example we considered last lecture - the bouncing tennis + basket balls example.

Recall the setup



We showed that conservation of energy means the basketball has downwards velocity $v_2 = \sqrt{2gh}$ before colliding with the earth. [cont...]

Assuming the Earth is infinitely massive, the basketball bounces upward with the same kinetic energy (speed!)

$\Rightarrow$ $v_{2i} = \sqrt{2gh}$ in diagram (2)

Assuming $h \gg$ diameter of the tennis ball and the basketball

$\Rightarrow$ $v_{1i} \simeq \sqrt{2gh}$ downwards (in diagram (2))

Then use our result from earlier

$$v_{1f} = \frac{2m_2 v_{2i} + (m_1 - m_2) v_{1i}}{m_1 + m_2}$$

$$= \frac{2 \cdot 10m \sqrt{2gh} + (m - 10m)(-\sqrt{2gh})}{m + 10m}$$

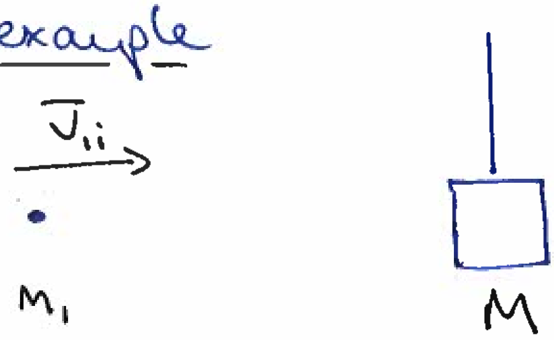
$$= \frac{(20m + 9m)}{11m} \sqrt{2gh}$$

$$= \frac{29}{11} \sqrt{2gh} \leftarrow \text{note } |v_{1i}| = \sqrt{2gh}$$

$$\simeq 2.6 \cdot |v_{1i}|$$ $\Rightarrow$ 2.6 times faster!

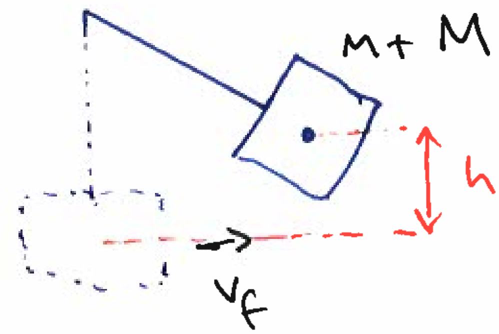
Pellet - block example

initial



really should draw 3 steps!

final



inelastic collision ⇒ momentum conserved
kinetic energy not conserved

$$\bar{p}_i = m\bar{v}_{ii} + 0 = m\bar{v}_{ii}$$

$$\bar{p}_f = (M+m)\bar{v}_f$$

We now apply conservation of energy to find height

$$E_i = E_f \Rightarrow E_{ik} + \cancel{E_{ip}}^{\,0} = \cancel{E_{fk}}^{\,0} + E_{fp}$$

$$\frac{1}{2}(M+m)v_f^2 = (M+m)gh$$

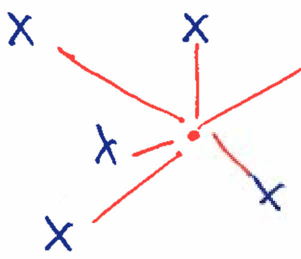
$$\Rightarrow v_f = \sqrt{2gh}$$

So use this in

$$m v_{ii} = (M+m)\, v_f$$

$$v_{ii} = \left(\frac{M+m}{m}\right) v_f = \left(\frac{M+m}{m}\right)\sqrt{2gh}$$

Centre of mass (slide 9)

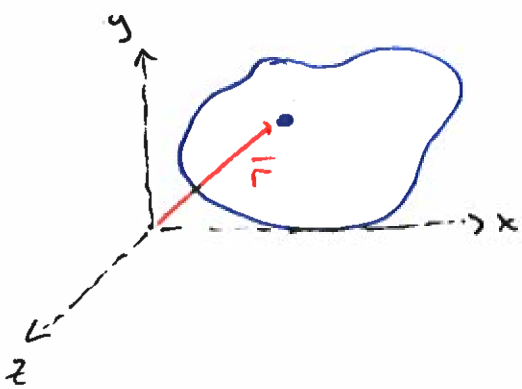


Coordinates of the centre of mass

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{m_1 + m_2 + \ldots + m_n}$$

$$= \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

$$y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{M} \qquad z_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$



Consider an infinitesimal mass (or volume of material) at a point at position $\bar{r}$

$$\sum_i m_i x_i \rightarrow \int dm_i \bar{r}_i$$

Don't forget to divide by the total mass!

$$\bar{r}_{cm} = \frac{1}{M} \int \bar{r}\, dm$$

We can define the <u>centre of mass motion</u> via

$$\bar{v}_{cm} = \frac{d\bar{r}_{cm}}{dt} \qquad \bar{a}_{cm} = \frac{d^2 \bar{r}_{cm}}{dt^2}$$

$$\hookrightarrow \bar{v}_{cm} = \frac{1}{M} \sum_{i=1}^{n} m_i \frac{d\bar{r}_i}{dt} = \frac{1}{M} \sum_{i=1}^{n} m_i \bar{v}_i = \frac{1}{M} \sum_{i=1}^{n} \bar{p}_i = \frac{1}{M} \bar{P}_{tot}$$

The "total momentum" is the "momentum of the centre of mass motion"

$$\bar{P}_{tot} = M\bar{v}_{cm} \quad (= \bar{P}_{cm})$$

This is the momentum of a particle of mass M, following the centre of mass motion (trajectory)

Conservation of momentum tells us that

$$\bar{P}_{cm,i} = \bar{P}_{cm,f}$$

Note that the centre of mass obeys

$$\bar{F}_{net} = \frac{d}{dt}\bar{P}_{cm} = M\bar{a}_{cm}$$