Physics 101H

General Physics 1 – Honors

Lecture 2 – 9/1/22

Problem-solving strategies

Welcome!

I am Prof. Monahan

Pronouns: **he/him/his**

Email**: cjmonahan@wm.edu**

Office: **Small Hall 326C**

Am I in the right room?

This course: for students who **intend to major in physics or physical sciences**

I assume you have a strong preparation in mathematics

- Comfortable with **calculus**

PHYSICS 101 (not this course!) does not assume knowledge of calculus

- Held at the same time
- Uses the same textbook
- Add/Drop deadline is **11:59 pm** on **Monday September 12**

DO NOT MISS THIS DEADLINE

Problem sets

Problem Set 0 has been posted on **Blackboard**

Due by the **start of class** (i.e. 10:59 am) on **Wednesday 7 September**

Remember

- I will **drop the lowest grade** on your weekly Problem Sets (but not including Problem Set 0)

Problem sets

Handwritten problems:

- Partial credit **is** available
- Graded by a graduate student grader

Hints:

- You will lose points if your solution is illegible or a mess
- Pen or pencil acceptable
- If you use a pen, cross through entire lines with a single line

MAKING SURE YOUR WORK IS LEGIBLE IS *YOUR* RESPONSIBILITY

Problem sets

Submitting handwritten problem sets:

- Produce a **single PDF** of your written notes
  - If you have a scanner, then that is the best option
  - Photos will work – use an app like Adobe Scan to make them legible
- Name your **single PDF file** `lastname_hwXX.pdf`
  - Replace "`lastname`" with your last name
  - Replace "`XX`" with the problem set number
- Submit your **single PDF file called** `lastname_hwXX.pdf` to **Blackboard**
  - Go to the appropriate assignment in Blackboard
  - Drag and drop your pdf file to the Attach Files section
  - Hit submit

Problem sets

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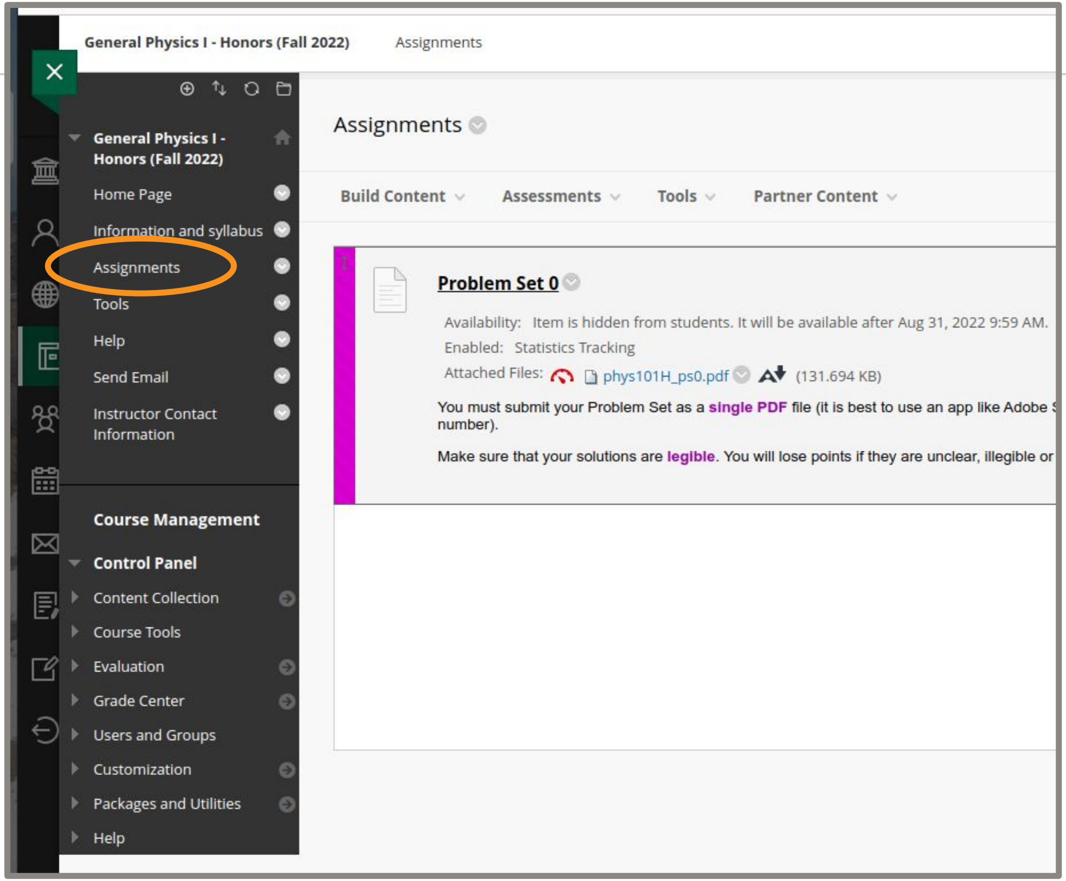


Undergraduate research: info sessions

Thursday, September 8th 11am–12pm
Thursday, September 15th 2–3 pm
Monday, September 19th 3:30–4:30 pm
Tuesday, October 4th 3–4 pm

Take place in Blow Hall 201

Undergraduate research can be done in any major and lead to some amazing opportunities – but how do you get started? At this info session, learn more about what undergraduate research entails, how to find a faculty mentor, and what funding opportunities exist. Open to all majors and class years. Snacks provided!

Summary

Topics

Yesterday: Course introduction

- Course details
- Grading and homework
- Logistics and timetable

Today: Problem solving [chapter 1]

- Basic strategies
- Setting up and solving problems
- Troubleshooting

What problem solving strategies do you use?

Tips and tricks

These tips and tricks may help with many problems, but are not a checklist you have to memorise and apply to every single problem

- Stuck on a problem? Revisit these notes – see if one helps unstick you
- Use these ideas to check your answer
- Keep practising problems and you will internalise many of these
- Not all strategies are necessary for all problems, but practise using them on simpler problems to reinforce their application when things get tougher (for example, in research)

Basic strategies: I

- Solve problems **symbolically**
  - Change all numbers to symbols
  - Solve equations for the unknown
  - Plug in numbers **at the end**
- Draw a **diagram**
  - Really. Draw a big diagram with labels.
- Check **units, dimensions and orders of magnitude**
  - May help determine the form of the solution
  - Can help you identify algebra mistakes

Example: Consider a mass on a string of length 1m, swinging through a small angular amplitude under only the force of gravity. What is the period of the swing?

Example [a “Fermi problem”]: How many people in the world are using their cell phones in any given minute?

Basic strategies: II

- Check **limiting and special cases**
- Check the **Taylor series expansion**

Useful Taylor Series

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} x^n = 1 - x + x^2 - x^3 + \dots$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} + \dots$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \dots$$

$$\arctan x = \sum_{n=0} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots$$

$$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n = 1 + mx + \frac{m(m+1)}{2} x^2 + \dots$$

Example: Show that the expression for nonrelativistic kinetic energy emerges as the low speed limit of the relativistic kinetic energy formula.

Specific strategies: set up

- Read the problem carefully
- Identify:
  - What you know
  - What you are trying to solve for
  - Extraneous information
- Identify:
  - Initial and final states
  - Constraints
- Choose a reference frame (choose wisely)
- Draw a big diagram with labels
- Try to stay organised

Specific strategies: solving

*Not really (I hope!), but it is a good way to remember to draw big diagrams and write out your solution attempts in full.

- Try to think in terms of physical statements, not equations
  - Identify the physical principles involved
  - Convert physical statements into equations
  - Make sure the number of equations = number of unknowns
- Draw a big diagram with labels
- Try to stay organised
  - Don't be afraid to use plenty of paper

REMEMBER: PHYSICISTS HATE TREES*

Specific strategies: troubleshooting

- Check the units of your numerical answer
- Check the size of your numerical answer is reasonable
- Reduce the problem to a simpler problem
  - Break it up into steps
  - Try to identify similar problems that you know how to solve
- Re-read the problem
- Check your maths, especially the minus signs
- Call on your intuition

Practice in pairs

*Explaining your attempts and/or your solutions helps you clarify your own understanding, as well as helping others learn. You may also learn new tips and techniques from your peers.

Instructions: Discuss the following question with a neighbour. Your answers will not be graded; your discussion is for your own learning*.

Question: The Schwarzschild radius R_S defines the ***event horizon*** of a black hole of mass m. The Schwarzschild radius depends on the speed of light c, and the gravitation constant G (with units $m^3/(kg\ s^2)$). Which of the following quantities is R_S proportional to?

(a) $G/(m\ c^2)$ (b) $G\ m/c^2$ (c) $G\ m/c^3$ (d) $c^2/(G\ m)$ (e) $c^3/(G\ m)$

Practice in pairs

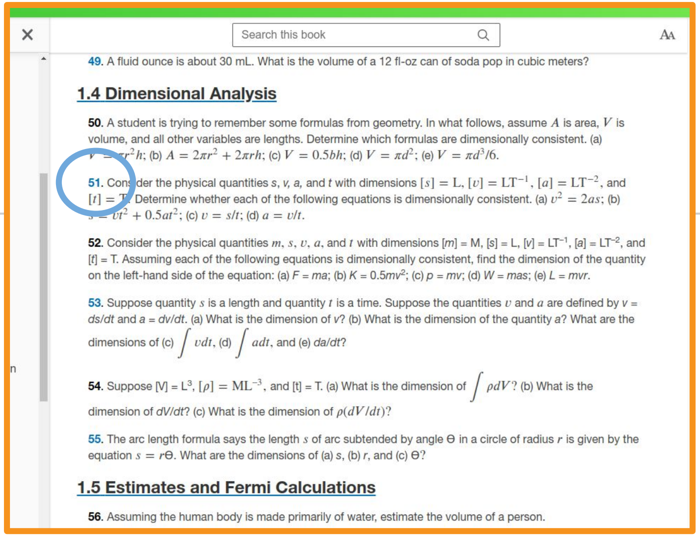
Question: Which of the following quantities is R_S proportional to?

(a) $G/(m\ c^2)$ (b) $G\ m/c^2$ (c) $G\ m/c^3$ (d) $c^2/(G\ m)$ (e) $c^3/(G\ m)$

Want more practice?

Check out the following problems in the textbook

- Conceptual questions: 9, 11, 13
- Problems: 14+15, 18, 19, 29, 31, 41, 51, 61, 65, 83



49. A fluid ounce is about 30 mL. What is the volume of a 12 fl-oz can of soda pop in cubic meters?

1.4 Dimensional Analysis

50. A student is trying to remember some formulas from geometry. In what follows, assume A is area, V is volume, and all other variables are lengths. Determine which formulas are dimensionally consistent. (a) $V = \pi r^2 h$; (b) $A = 2\pi r^2 + 2\pi rh$; (c) $V = 0.5bh$; (d) $V = \pi d^2$; (e) $V = \pi d^3/6$.

51. Consider the physical quantities s, v, a, and t with dimensions $[s] = \mathrm{L}$, $[v] = \mathrm{LT}^{-1}$, $[a] = \mathrm{LT}^{-2}$, and $[t] = \mathrm{T}$. Determine whether each of the following equations is dimensionally consistent. (a) $v^2 = 2as$; (b) $s = vt^2 + 0.5at^2$; (c) $v = s/t$; (d) $a = v/t$.

52. Consider the physical quantities m, s, v, a, and t with dimensions $[m] = \mathrm{M}$, $[s] = \mathrm{L}$, $[v] = \mathrm{LT}^{-1}$, $[a] = \mathrm{LT}^{-2}$, and $[t] = \mathrm{T}$. Assuming each of the following equations is dimensionally consistent, find the dimension of the quantity on the left-hand side of the equation: (a) $F = ma$; (b) $K = 0.5mv^2$; (c) $p = mv$; (d) $W = mas$; (e) $L = mvr$.

53. Suppose quantity s is a length and quantity t is a time. Suppose the quantities v and a are defined by $v = ds/dt$ and $a = dv/dt$. (a) What is the dimension of v? (b) What is the dimension of the quantity a? What are the dimensions of (c) $\int v dt$, (d) $\int a dt$, and (e) da/dt?

54. Suppose $[V] = \mathrm{L}^3$, $[\rho] = \mathrm{ML}^{-3}$, and $[t] = \mathrm{T}$. (a) What is the dimension of $\int \rho dV$? (b) What is the dimension of dV/dt? (c) What is the dimension of $\rho(dV/dt)$?

55. The arc length formula says the length s of arc subtended by angle Θ in a circle of radius r is given by the equation $s = r\Theta$. What are the dimensions of (a) s, (b) r, and (c) Θ?

1.5 Estimates and Fermi Calculations

56. Assuming the human body is made primarily of water, estimate the volume of a person.

For each topic, I will try to suggest good ungraded practice problems

Note that answers are provided for questions with blue numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

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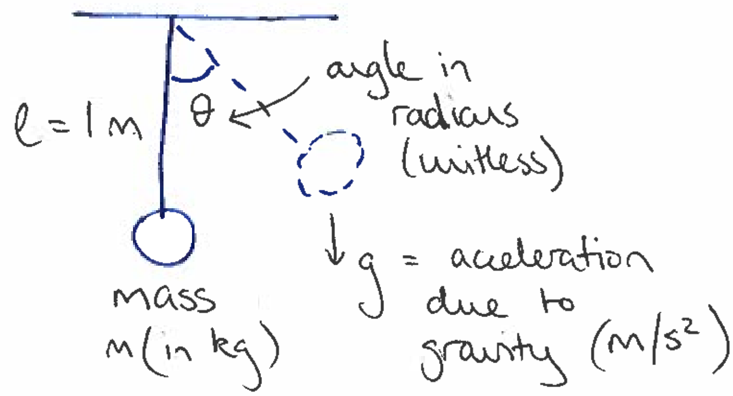
Pendulum example:

Step 1 - draw a diagram



Step 2 - identify what we need to solve for

Period = time (units = s)

Step 3 - try combining units

N.B. formula cannot depend on mass, because there is nothing to cancel "kg"

$$T \propto \sqrt{\frac{\ell}{g}} = f(\theta)\sqrt{\frac{\ell}{g}}$$

"Fermi" problem:

Guess ~ 1/2 people in the world have cell phones

~ 7×10^9 people (7 billion) ⇒ 3.5×10^9 phones

↑ round up to 4×10^9

Guess ~ use cell phone for 4 hours a day ← average American uses 5.4 hrs!

⇒ $\frac{4}{24} = \frac{1}{6}$ of the minutes in a day

So $4 \times 10^9 \times \frac{1}{6} = \frac{2}{3} \times 10^9$ people or $\boxed{\sim 7 \times 10^8 \text{ people}}$

↖ $2/3 = 0.666\ldots \sim 0.7$

Taylor series (slide 14):

derivative with respect to x — powers of (x-a)

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

"Taylor series of f(x) around x = a"

$n! = n \cdot (n-1) \cdot \dots \cdot (2) \cdot (1)$

If x-a is small then first few terms are a good approximation

Coefficients are different derivatives of f evaluated at a, divided by n!

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Relativistic kinetic energy:

Recall nonrelativistic kinetic energy $T_{NR} = \frac{1}{2}mv^2$

In special relativity $T_R = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$!

Consider $v^2/c^2 \ll 1$ and write $\frac{1}{\sqrt{1-v^2/c^2}} = (1-v^2/c^2)^{-1/2}$

Expand using Taylor series

$$(1+x)^{-m} = 1 - mx + \dots$$

$$\Rightarrow (1-v^2/c^2)^{-1/2} = 1 - \frac{1}{2}\left(-\frac{v^2}{c^2}\right) + \dots = 1 + \frac{1}{2}v^2/c^2 + \dots$$

So $T_R \simeq mc^2\left[\left(1 + \frac{1}{2}v^2/c^2 + \dots\right) - 1\right] = \frac{1}{2}mv^2$ if $v^2/c^2 \ll 1$ or $v^2 \ll c^2$