Physics 101H

General Physics 1 – Honors

Lecture 16 – 9/29/22

Work and energy

Quick quiz

*Quick quizzes incorporate *retrieval practice* and *interleaving*, in which we revisit older material to reinforce your understanding. By keeping track of answers that you can and can't write down without reference to your notes, these quizzes help you identify which topics and concepts you understand best and which you may need to keep reinforcing.

Instructions: This quiz is for your own learning. There are three questions and each question has two columns. Write your own solution, without reference to your notes, the textbook, or your neighbour, **in the first column**. Once you have tried to answer all the questions, discuss the questions with a neighbour and fill in any incomplete answers **in the second column**. Keep your sheet for future reference.

Summary

Topics

Wednesday: Work [chapter 7]
- Work done
- Constant and varying forces
- Conservative forces

Today: Work & energy [chapters 7/8]
- Work-energy theorem
- Potential energy

Next week:
- Midterm
- Drag & computational physics
- Energy conservation

Announcements **This week:** **First midterm on Mon October 3**

Work-energy theorem

Let's consider the work done by a varying force along a wavy path

Work-energy theorem: net work done on an object by external conservative forces equals the change in the object's kinetic energy

Which is more work - lifting an object directly to a given height, or pushing it up a smooth (i.e. frictionless) incline plane to the same height?

What about if the plane is rough (i.e. with friction)?

Potential energy

When we consider more complicated systems things get more interesting

- Multiple particles
- Internal forces

Now we can do more with our work:change

- Change the kinetic energy of the constituents
- Change the internal configuration of the system

Changing the configuration of the system stores work as **potential energy**

Potential energy requires a reference system to make sense!

- Only changes in potential energy matter

Example: What is the work done when compressing a spring?

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PHYSICS 101 - HONORS

Lecture 16 9/30/22

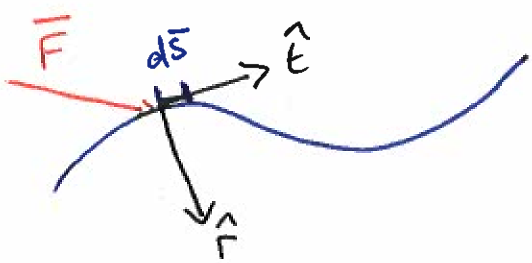
Work-energy theorem (slide 4)

Let's go back to our definition of work, in the most general case (varying force, wavy path)

$$W = \int \bar{F} \cdot d\bar{s}$$

How can we determine this for an arbitrary path?

Start with a diagram!



Choose a reference frame
$\hat{t}$ = unit vector tangent to curve
$\hat{r}$ = unit vector radially

$\Rightarrow d\bar{s} = ds\,\hat{t}$

Our force in this frame is

$$\bar{F} = F_r\,\hat{r} + F_t\,\hat{t}$$

We can write

$$W = \int dW$$

← sum over infinitesimal bits of work done

$$\Rightarrow dW = \bar{F} \cdot d\bar{s}$$
$$= (F_r\,\hat{r} + F_t\,\hat{t}) \cdot (ds\,\hat{t})$$
$$= F_r \underbrace{(\hat{r} \cdot \hat{t})}_{=0} ds + F_t\, ds\, \underbrace{\hat{t} \cdot \hat{t}}_{=1} = F_t\, ds$$

But $F_t = m a_t$ $\qquad (\bar{a} = a_r \hat{r} + a_t \hat{t})$

$= m \frac{dv}{dt}$

So $dW = F_t \, dS = m \frac{dv}{dt} dS$

By the chain rule

$$\frac{dv}{dt} = \frac{dv}{dS} \underbrace{\frac{dS}{dt}}_{v} = \frac{dv}{dS} v \quad \Rightarrow \quad \frac{dv}{dt} = v \frac{dv}{dS} \, dS = v \, dv$$

Thus $dW = m \frac{dv}{dt} = m v \, dv$

And $W = \int dW = \int m v \, dv$

$$= m \int_{v_1}^{v_2} v \, dv = m \frac{v^2}{2} \Big|_{v_1}^{v_2} = m \left(\frac{v_2^2}{2} - \frac{v_1^2}{2} \right)$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \Delta K \qquad !!!$$

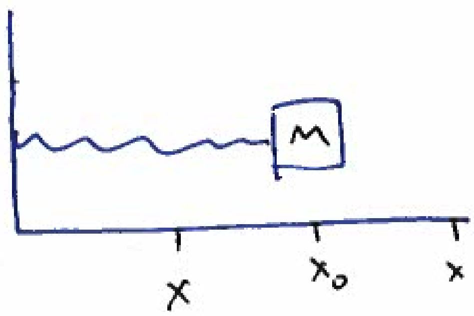
This is the work-energy theorem:

for a one particle system, the work done by a conservative force is equal to the change in kinetic energy of the system.

Compressing a spring (slide 8)

Recall from lecture 14 : $W = \frac{k}{2}(x_1^2 - x_2^2)$

Go back to our diagram



If we treat the spring as part of the system, then compressing or extending the spring stores potential energy in the system.

$$W_s = -W_{ext} = \frac{k}{2}(x_2^2 - x_1^2)$$

$$= \frac{kx_2^2}{2} - \frac{k}{2}x_1^2 = \Delta U$$ ← denote potential energy by U

($\frac{kx_2^2}{2}$: elastic potential energy)

Potential energy more generally

For conservative forces: $W_{int} = \int_{x_1}^{x_2} \bar{F} \cdot d\bar{x} = -\Delta U$

$$= -U \Big|_{x_1}^{x_2} \Rightarrow F_x = -\frac{dU}{dx}$$

More generally

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z} \qquad \bar{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$$

$$\Rightarrow \bar{F} = -\frac{\partial U}{\partial x}\hat{x} - \frac{\partial U}{\partial y}\hat{y} - \frac{\partial U}{\partial z}\hat{z} = -\bar{\nabla} U$$ ← gradient