Physics 101H

General Physics 1 – Honors

Lecture 15 – 9/29/22

Part I review

Midterm 1

Good News: No problem set assigned on Wednesday!

Bad News: First midterm will take place on **Monday October 3**!

You will have 45 minutes to complete the exam

- 3 multiple choice questions
- 2 handwritten solution problems

Bring paper and something(s) to write with! (Spare paper will be available)

Topics cover Chapters 1 to 6 and include:

- Vectors
- 1D and 2D kinematics
- Newton's laws of motion

No questions on *Motion in a medium* or *Noninertial frames*

You may prepare your own formula sheet – **one side** of **letter paper**

You may bring a calculator, but phones, tablets and laptops are not allowed

Remember you are here to learn and understand the physics!

Topic overview

Preliminaries:

- Dimensional analysis and units
- Vectors, including dot and cross products

1D kinematics:

- Distinguishing scalar and vector quantities
- Kinematic equations for constant acceleration

2D kinematics:

- Kinematic equations for constant acceleration
    - Projectile motion
- Uniform circular motion

Dynamics

- Newton's laws of motion

Example: A car and a motorcycle are racing. To give the car a fair chance, the motorcycle starts halfway down the race track and must wait until the car passes before starting. At the instant the car passes the motorcycle, the motorcycle accelerates with constant acceleration and starts to race. What is the speed of the motorcycle when it catches up to the car? Assume that the car travels with constant speed. Hint: Draw the speed vs. time plots on top of each other.

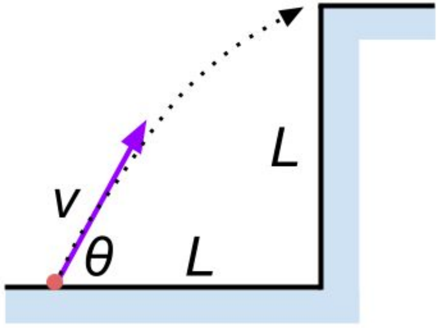
(a) v (b) 3v/2 (c) 2v (d) 3v (e) 4v

Example: Which of the following quantities could be a force [l is a length, m is a mass, a is an acceleration, v is a speed and t is a time]?

(a) mv^2 (b) $m\,a\,t$ (c) mv/t (d) mv/l (e) v^2/l

Example: Find the maximum speed that a car can travel around a curve with radius of curvature R = 20.0 m without slipping. Assume that the coefficient of static friction is 1.00.

Example: A ball is thrown at an angle θ up to the top of a cliff of height L, from a point a distance L from the base. What initial speed causes the ball to land right at the edge of the cliff?



Example: A projectile is launched at an angle of 30° and lands 20 s later at the same height as it was launched. (a) What is the initial speed of the projectile? (b) What is the maximum altitude? (c) What is the range?

Example: A machine at a post office sends packages out a chute and down a ramp to be loaded into delivery vehicles. (a) Calculate the acceleration of a box heading down a 10.0° slope, assuming the coefficient of friction for a parcel on waxed wood is 0.100. (b) Find the angle of the slope down which this box could move at a constant velocity. You can neglect air resistance in both parts.

Studying for midterm 1

Good luck!

Studying for the midterm:

- Look over Problem Sets
- Work through examples from class and in the textbook

When working through problems (especially someone else's solution):

- Cover up the solution and try to work out the next step in the solution
- If you can't figure that out, uncover just the first step and then try to figure out the next steps
- Try to *self-explain*, that is – write down your thought process and what principles, concepts or equations are being applied at each step.

Remember that you are here to learn and understand the physics!

[But also remember there are two methods for calculating your final grade]

PHYSICS 10H - HONORS

Lecture 15 9/29/22

Key equations (slide 5)

Vectors : $\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}|\cos\theta$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Kinematic equations :
(constant acceleration)

$$x = \frac{at^2}{2} + V_0 t + x_0$$

$$v = v_0 + at$$

$$\Delta x = \left(\frac{v + v_0}{2}\right) t$$

$$v^2 = v_0^2 + 2a\Delta x$$

Projectile motion :

$$g = 9.81 \ m/s^2$$

$$h = \frac{v_0^2}{2g} \sin^2\theta$$

$$R = \frac{2v_0^2}{g} \sin\theta \cos\theta$$

Uniform circular motion :

$$a_c = \frac{v^2}{r} = \omega^2 r \qquad T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

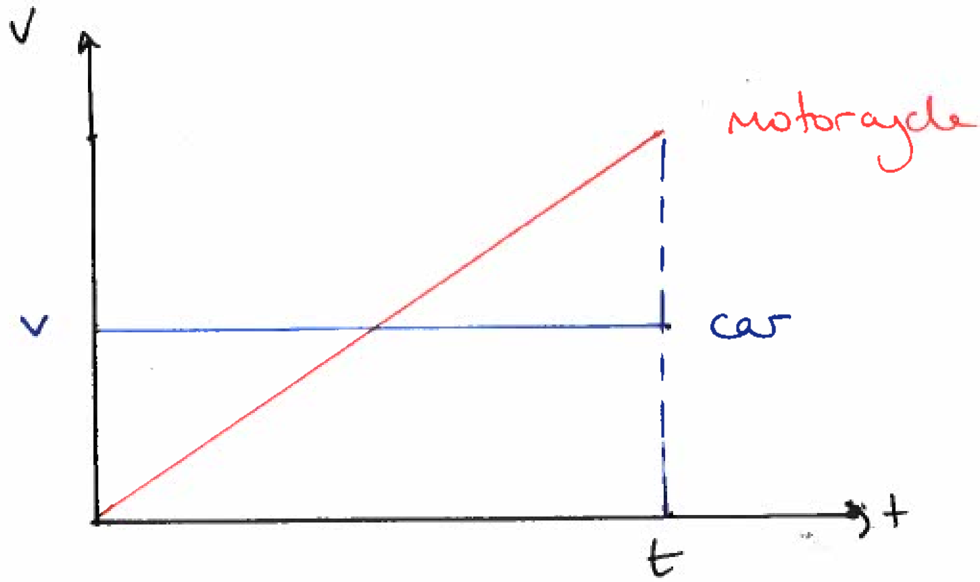
$$v = \omega r$$

Newton's 2nd law :

$$\bar{F}_{net} = \sum_i \bar{F}_i = m\bar{a} \qquad F_s \leq \mu_s N$$

$$F_k = \mu_k N$$

Car and motorcycle example



Distance travelled by car = area under speed "curve"

$= v \cdot t$

Distance travelled by motorcycle = area under triangle

$= \frac{1}{2} v_M t$

$vt = \frac{1}{2} v_M t \Rightarrow \boxed{v_M = 2v}$ answer (c)

Units example

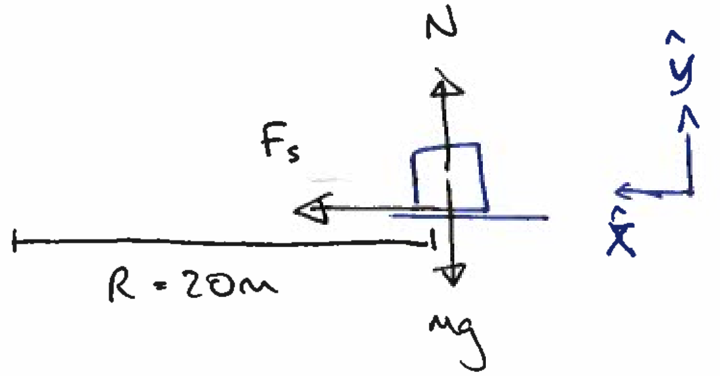
(a) = energy (b) ma is a force ⇒ force × time (an impulse)

(c) = kg m/s /s (d) kg/s (e) acceleration !

$= \boxed{\text{kg m/s}^2}$ ✓

answer (c)

Car and curve example



Vertically: $F_{net} = N - mg = m \cdot 0 = 0$ ($a_{vert} = 0$)

$\Rightarrow N = mg$

radially: $F_{net} = F_s = m a_c$

But $F_s = \mu_s N = \mu_s mg$

$\Rightarrow \mu_s mg = m a_c$

$\Rightarrow a_c = \mu_s g = g$ ($\mu_s = 1$ in question)

Uniform circular motion $\Rightarrow a_c = \frac{v^2}{R}$

We want v, so combining these gives

$\frac{v^2}{R} = g$ or $v_{max} = \sqrt{g \cdot R}$

$= \sqrt{9.81 \cdot 20.0}$

$= \boxed{14.0 \text{ m/s}}$

Cliff example

In the horizontal direction:

$x_0 = 0 \quad x = L \quad t = ? \quad v_{0x} = v_0 \cos\theta \quad a = 0$

$x = \frac{1}{2}(v + v_0)t + x_0 \qquad v = v_0 \quad \text{since } v = v_0 + at = v_0$

$\left(\text{or} \quad x = v_x t \quad \text{for } a = 0\right)$

$t = \frac{x}{v_x} = \frac{L}{v_0 \cos\theta} \qquad (*)$

In the vertical direction:

$y_0 = 0 \quad y = L \quad t = ? \quad v_{0y} = v_0 \sin\theta \quad a = -g \quad v = ?$

$y = \frac{1}{2}at^2 + v_{0y}t + y_0$

$L = -\frac{g}{2}t^2 + v_0 \sin\theta \, t + 0 \qquad (**)$

Use $(*)$ in $(**)$ $\Rightarrow$ $L = -\frac{g}{2}\left(\frac{L}{v_0\cos\theta}\right)^2 + v_0\sin\theta\left(\frac{L}{v_0\cos\theta}\right)$

or $L = \frac{-gL^2}{2v_0^2\cos^2\theta} + L\tan\theta$

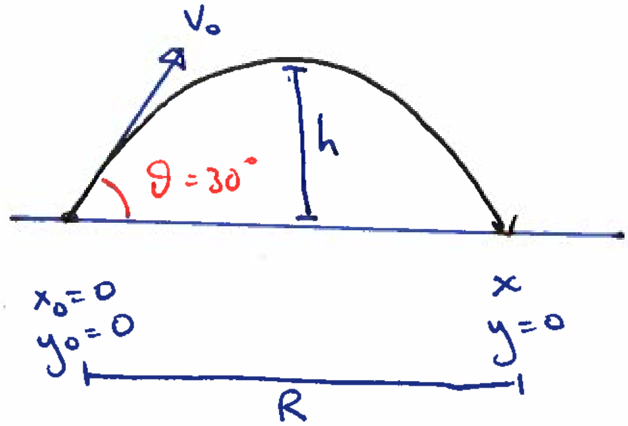
$\Rightarrow 2\cos^2\theta \, L \, v_0^2 = -gL^2 + 2Lv_0^2\cos^2\theta \tan\theta$

$v_0^2\left(2L\cos^2\theta - 2L\cos^2\theta\tan\theta\right) = -gL^2$

$v_0^2 \cdot 2L\cos\theta\left(\cos\theta - \cos\theta\tan\theta\right) = -gL^2$

$v_0^2 = \frac{-gL^2}{2L\cos\theta\left(\cos\theta - \sin\theta\right)} \quad \Rightarrow \quad \boxed{v_0 = \sqrt{\frac{gL}{2\cos\theta\left(\sin\theta - \cos\theta\right)}}}$

Projectile example



horizontally:

$x_0 = 0, \; x = ?, \quad t = 20\,s$

$v_{0x} = v_0 \cos\vartheta \quad a = 0$

$\Rightarrow v_x = v_{0x}$

vertically:

$y_0 = 0, \; y = 0, \quad t = 20\,s$

$v_{0y} = v_0 \sin\vartheta, \quad a = -g$

$v_y = -v_0 \sin\vartheta$

a) Use $v = v_0 + at$ vertically

$\Rightarrow -v_0 \sin\vartheta = v_0 \sin\vartheta - gt$

$\Rightarrow 2 v_0 \sin\vartheta = gt$

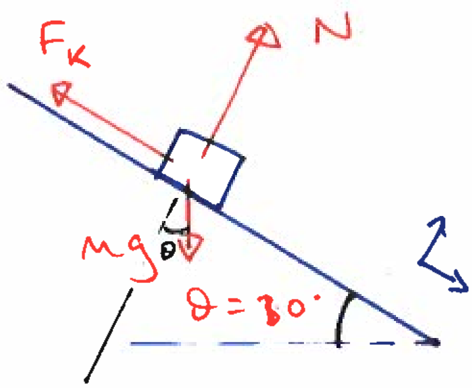
$$v_0 = \frac{gt}{2\sin\vartheta} = \frac{9.81 \cdot 20}{2 \cdot \sin 30°} = \boxed{196.2\ m/s}$$

(b) $h = \dfrac{v_0^2}{2g}\sin^2\vartheta = \dfrac{196.2^2}{2 \cdot 9.81} \cdot \sin^2 30° = \boxed{490.5\ m}$

(c) $x = v_{0x} t = 196.2 \cdot 20 = \boxed{3924\ m}$

Package example

(a)



$\theta = 10^\circ \qquad \mu_K = 0.100$

⊥ to slope:

$F_{net} = ma$

$\Rightarrow N - mg\cos\theta = 0$

$\Rightarrow N = mg\cos\theta$

∥ to slope:

$F_{net} = ma$

$\Rightarrow mg\sin\theta - F_K = ma$

Use $F_K = \mu_K N$

$= \mu_K mg\cos\theta$

$\Rightarrow mg\sin\theta - \mu_K mg\cos\theta = a$

or $a = g(\sin\theta - \mu_K\cos\theta)$

$= 9.81(\sin 10^\circ - 0.100\cos 10^\circ)$

$= \boxed{0.737\ m/s^2}$

(b) Let new angle $= \alpha$. Constant velocity $\Rightarrow a_{\parallel} = 0$

$\Rightarrow mg\sin\alpha - \mu_K\cos\alpha\, mg = 0$

or $\sin\alpha = \mu_K\cos\alpha$

$\Rightarrow \tan\alpha = \mu_K \quad \Rightarrow \quad \alpha = \arctan\mu_K = \arctan 0.100$

$= \boxed{5.71^\circ}$