Physics 101H

General Physics 1 – Honors

Lecture 13 – 9/23/22

Forces

Summary

Topics

Wednesday: Noninertial frames

- Noninertial reference frames
- Fictitious/pseudo forces
  - Centrifugal “force”
  - Coriolis “force”

Today: Motion through a medium

- Motion through a medium
- Models of resistance
  - Linear
  - Quadratic

This week:

- Motion through a medium
- Work, energy & power
- Review

Announcements

This week: **No problem set this week!**
First midterm on Mon October 3

Two minute essay

Instructions: Write one paragraph on the following topic. You have two minutes. You may not use your notes and you should not consult with others around you. Your answer will not be graded; your answer is for your own learning and you don't need to share your answer.*

Question: Revisit your two minute essay on what happens if you hold a pendulum that is free to swing (such as a shoe on a shoestring) inside a plane accelerating down a runway during takeoff. [Remember that from Lecture 11 on Monday 19 September?!] Explain your reasoning in light of our discussion of fictitious forces on Wednesday.

Problem solving in pairs

Instructions: Attempt the following question with a neighbour. Your answers will not be graded; your discussion is for your own learning. It is ok if you do not complete the question, but make sure you identify the key steps and write down the main equations.

Question: Two objects are connected by a light string that passes over a frictionless pulley. One object hangs from the string vertically below the pulley and the other lies on a frictionless incline plane. Find: (a) the magnitude of the acceleration of the objects; and (b) the tension in the string.

Motion through a medium

Many of our examples specify “frictionless” planes and pulleys and so on

But real objects experience friction when moving through a medium

- For example: air drag or viscosity
- Resistive force due to the medium
- Opposes the relative motion of the object and the medium
- Magnitude of the resistive force depends on the relative speed, possibly in some complicated (nonlinear) way

Motion through a medium

Resistance model

At low speeds, we can approximate the resistive force as linear in the speed

Leads to **terminal velocity** – constant velocity at which the projectile travels

At higher speeds, we can model the resistive force as quadratic in the speed

NEXT WEEK:
THE FIRST MIDTERM IS ON MONDAY OCTOBER 3

Midterm 1

Good News: No problem set assigned on Wednesday!

Bad News: First midterm will take place on **Monday October 3**!

You will have 45 minutes to complete the exam

- 3 multiple choice questions
- 2 handwritten solution problems

Bring paper and something(s) to write with! (Spare paper will be available)

Topics cover Chapters 1 to 6 and include:

- Vectors
- 1D and 2D kinematics
- Newton's laws of motion

No questions on *Motion in a medium* or *Noninertial frames*

You may prepare your own formula sheet – **one side** of **letter paper**

You may bring a calculator, but phones, tablets and laptops are not allowed

Remember you are here to learn and understand the physics!

Summary

Topics

Today: Motion through a medium

- Motion through a medium
- Models of resistance
  - Linear
  - Quadratic

Wednesday: Work [chap. 7]

- Work done
- Constant force
- Varying force

This week:

- Motion through a medium
- Work, energy & power
- Review

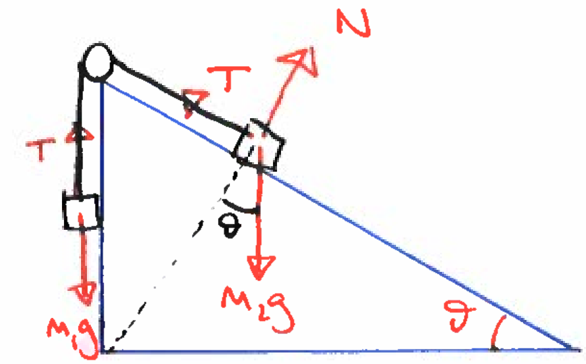
Announcements

This week: **No problem set this week!**
First midterm on Mon October 3

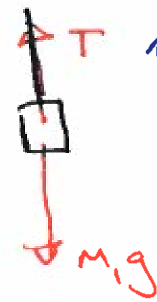
PHYSICS 101 - HONORS

Lecture 13   9/26/22

Pulley example



Force diagram M_1 :



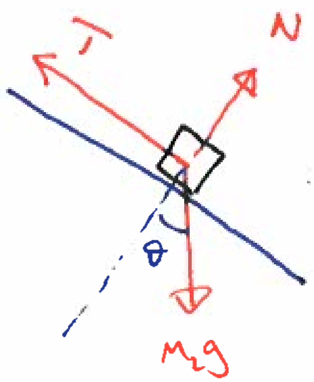
+ve y direction for M_1:

$$T - M_1 g = M_1 a \quad (1)$$

assume M_1 accelerates up !

$$\Rightarrow T = M_1(a+g)$$

Force diagram M_2 :



M_2 accelerates down slope
↓

Parallel to plane (+ve x) $\quad T - M_2 g \sin\theta = -M_2 a \quad (*)$

Perpendicular to plane $\quad M_2 g \cos\theta - N = 0$

Use $(*) \Rightarrow \quad T = M_2(-a + g\sin\theta) \quad (2)$

Now we have two equations in two unknowns !

Set (1) = (2) $\Rightarrow M_1(a+g) = M_2(-a + g\sin\theta)$

$$\Rightarrow M_1 a + M_2 a = -M_1 g + M_2 g \sin\theta$$

$$\Rightarrow a(m_1 + m_2) = m_2 g \sin\theta - m_1 g$$

$$\boxed{a = \frac{(m_2 \sin\theta - m_1)\, g}{m_1 + m_2}}$$

Now use this result in (1)

$$T = m_1 (a + g)$$

$$= m_1 \cdot \frac{(m_2 \sin\theta - m_1)\, g}{m_1 + m_2} + m_1 g$$

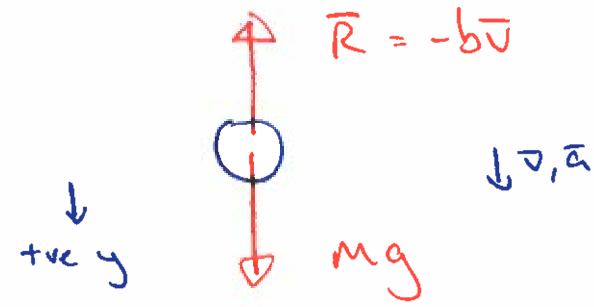
$$= m_1 g \left(1 + \frac{m_2 \sin\theta - m_1}{m_1 + m_2}\right)$$

$$= m_1 g \left(\frac{m_1 + m_2 + m_2 \sin\theta - m_1}{m_1 + m_2}\right)$$

$$= \boxed{\frac{m_1 m_2 g}{m_1 + m_2} (\sin\theta + 1)}$$

Linear resistance model (slide 6)

Assume $\bar{R} = -b\bar{v}$ $\Rightarrow$



1D motion with air resistance!

$$\bar{F}_g + \bar{R} = m\bar{a}$$

$\nwarrow$ careful with signs!
$\bar{F}_{net} = \bar{F} + \bar{R}$ = sum of all forces

$mg - bv = ma$ in y direction

$$a = g - \frac{b}{m}v$$
$$= -\frac{b}{m}\left(v - \frac{mg}{b}\right)$$

Write this as an ordinary differential equation (ODE)

$$\frac{dv}{dt} = -\frac{b}{m}\left(v - \frac{mg}{b}\right)$$

$$\Rightarrow \int \frac{dv}{v - \frac{mg}{b}} = -\frac{b}{m}\int dt$$

$$\Rightarrow \ln\left(v - \frac{mg}{b}\right) = -\frac{b}{m}t + c$$

$$\Rightarrow e^{\ln\left(v - \frac{mg}{b}\right)} = e^{-bt/m + c}$$

$D = e^{-c}$ arbitrary constant!

$$v - \frac{mg}{b} = De^{-bt/m}$$

$$v = De^{-bt/m} + \frac{mg}{b} = \frac{mg}{b}\left(1 + Ae^{-bt/m}\right)$$

more arbitrary constants! $D = \frac{mgA}{b}$ or $A = \frac{bD}{mg}$

Let's try to determine A, assuming some boundary conditions. Let's assume that at $t=0$, $v=0$. Then

$$v=0 \Rightarrow v(t=0) = \frac{mg}{b}\left(1 + A e^{-b\cdot 0/m}\right) = 0$$

or $\frac{mg}{b}(1+A) = 0 \Rightarrow 1 + A = 0$ because $\frac{mg}{b} \neq 0$

So $A = -1$

$$\Rightarrow v(t) = \frac{mg}{b}\left(1 - e^{-bt/m}\right)$$

Note that as $t \to \infty$

$$v(t \to \infty) \to \frac{mg}{b}\left(1 - \underbrace{e^{-b\infty/m}}_{\to 0}\right) = \frac{mg}{b}$$

Velocity tends to a constant, called the terminal velocity

Quadratic resistance model (slide 7)

Take $\bar{R} = -\frac{1}{2} D \rho A v^2 \hat{v}$

- $-$: antiparallel to $\hat{v}$
- D: drag coefficient
- ρ: density of medium
- A: area of projectile
- v^2: quadratic in v (ie proportional to $|\bar{v}|^2$)

Now we have $\bar{F}_g + \bar{R} = m\bar{a}$

In 1D $mg - \frac{D}{2}\rho A v^2 = ma$

Terminal velocity requires $a = 0$

$$\Rightarrow mg - \frac{D}{2}\rho A v^2 = 0$$

or $$v^2 = \frac{2mg}{D\rho A}$$

$$\Rightarrow v_T = \sqrt{\frac{2mg}{D\rho A}}$$

But what about the full ODE?

$$m\frac{dv}{dt} = mg - \frac{D\rho A}{2}v^2$$

This is much more complicated, but we can solve it. In 2D, it gets much more interesting! We will revisit this in a problem set soon!